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Dong-Huyn Oh

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Donghyun Oh*

*: Tel. +46-8-790-67 93, Fax. +46-8-790-95 17, Center of Excellece for Science and Innovation Studies, Royal Institute of Technology, SE-100 44, Sweden, *email*: donghoh@infra.kth.se

Abstract

The aim of this paper is to introduce an alternative measure that incorporates the concept of a global Malmquist productivity index with a directional distance function. Unlike a conventional geometric mean form of the Malmquist-Luenberger productivity index, this global index is circular and free from linear programming (LP) infeasibility. It can also be decomposed into certain sources of productivity growth such as efficiency change and technical change. The suggested methodology is employed in the analysis of 28 OECD countries over the period of 1990-2004. The empirical result shows that the northern European countries are found to have a higher productivity growth rate when compared with the rest of the OECD countries. Furthermore, we found that the main source of the productivity growth is technical change.

Keywords: Global Malmquist-Luenberger productivity index , Directional distance function , Productivity , Circularity
***JEL Classification*:** C43 , O47 , Q53

1. Introduction

Although productivity is not the only measure of economic prosperity, standard of living, and competitiveness of the economy, it has long been widely recognized as an indirect measure for them. As international concern over the sustainable growth increases, recent attempts have been made to develop measures of productivity growth that incorporate the negative effect of environmentally harmful outputs. The reason for these recent developments is that productivity measures are often biased if measured without the environmental effect, resulting in misleading policy prescriptions. A Malmquist productivity index, a distinguished non-parametric approach for measuring productivity growth, was modified by [4] to measure *environmental* productivity growth, and named the Malmquist-Luenberger productivity index (hereafter, ML index), which integrates the concepts of the Malmquist productivity index and a directional distance function. Ever since this seminal work, the ML index has been widely used in measuring the performance of a wide range of decision-making units (DMUs), such as utilities [1], manufacturing industries [14], the public sector [19], and states [20, 12, 9, 6]. One reason it has been widely exploited in empirical studies is that it can be decomposed into sources of productivity growth, which can help policy makers identify ways of increasing productivity.

Because the conventional Malmquist-Luenberger productivity index uses a geometric mean form of two contemporaneous ML indexes, ML index faces a potential LP infeasibility problem in measuring cross-period directional distance functions. Furthermore, a geometric mean form of two contemporaneous ML indexes is not circular[1]. Therefore, productivity

---

[1]Circularity, or transitivity, of an index is defined as follows: $I^{1,3} = I^{1,2} \times I^{2,3}$, where $I^{1,2}$, $I^{2,3}$, and $I^{1,3}$ represent the index between periods 1 and 2, 2 and 3, and 1 and 3, respectively.

growth measured using two adjacent ML indexes should be interpreted with caution. These shortcomings of the conventional ML index could provide policy makers with misleading measures of productivity growth.

The aim of this paper is to introduce an alternative methodology that can deal with multi-output and multi-input as well as environmentally bad outputs while avoiding the aforementioned disadvantages of the conventional ML index. This alternative methodology not only can measure the productivity growth of DMUs but also can be decomposed into the sources of the productivity growth, such as efficiency change and technical change. To develop our methodology, the concept of a global Malmquist productivity [16] and that of a directional distance function are combined. This integrated methodology is free from the LP infeasibility problem and is circular. The first step is to define a global production technology set and a contemporaneous production technology set in which good outputs as well as bad outputs are assumed to be produced by the inputs. By exploiting these two production technology sets, our alternative methodology, a global Malmquist-Luenberger productivity index (hereafter, GML index), is developed.

The methodology is applied to the evaluation of the productivity performance of 28 OECD countries over the period 1990-2004. Alghough several studies apply the ML index to the international and interregional analysis of productivity performance at the country level [12,18,6], they use the geometric mean form of the two contemporaneous ML indexes not the GML index. Our empirical results indicate that the northern European countries have higher productivity growth than other OECD countries, and that the main source of the productivity growth is technical change.

This paper proceeds as follows. In Section 2, we introduce an alternative Malmquist-Luenberger productivity index based on the concept of the global Malmquist productivity index and the directional distance function. Section 3 applies the methodology to the analysis of the 28 OECD countries. Section 4 presents our conclusions.

2. Methodology

The production technology for countries producing M outputs, $\mathbf{y} \in R_+^M$, and J polluting by-products, $\mathbf{b} \in R_+^J$, is represented by the output set $\mathbf{P}(\mathbf{x})$, which designates the set of good and bad outputs vector $(\mathbf{y}, \mathbf{b})$ that is jointly produced from N input vector, $\mathbf{x} \in R_+^N$. The production technology can be mathematically expressed as follows:

$$\mathbf{P}(\mathbf{x}) = \{(\mathbf{y}, \mathbf{b}) \,|\, \mathbf{x} \text{ can produce } (\mathbf{y}, \mathbf{b})\}. \tag{1}$$

To describe and model the production technology when desirable and undesirable outputs are jointly produced, a number of assumptions are needed in the form of axioms. These axioms let us use the methodology developed in this study to measure productivity growth of countries. The first assumption we make concerning the output set is that it is compact for each input vector $\mathbf{x} \in R_+^N$. Inputs are also assumed to be strongly or freely disposable, so

$$\text{if } \mathbf{x}' \geq \mathbf{x} \text{ then } \mathbf{P}(\mathbf{x}') \supseteq \mathbf{P}(\mathbf{x}). \tag{2}$$

Equation (2) means that the output set will not shrink if the inputs are increased.

Incorporation of undesirable outputs into the classical production technology requires the assumption of null-jointness, which implies that the decision makers (in this study, countries) should produce the undesirable outputs if they produce the desirable outputs. The assumption of null-jointness is expressed as follows:

$$\text{if } (\mathbf{y}, \mathbf{b}) \in \mathbf{P}(\mathbf{x}) \text{ and } \mathbf{b} = 0 \text{ then } \mathbf{y} = 0. \tag{3}$$

Equation (3) says that the desirable outputs cannot be produced if the undesirable outputs is produced when the null-jointness assumption is imposed into the production technology.

Weak disposability assumption is also introduced into the production technology set, which is mathematically stated as follows:

$$\text{if } (\mathbf{y},\mathbf{b}) \in \mathbf{P}(\mathbf{x}) \text{ and } 0 \le \theta \le 1 \text{ then } (\theta\mathbf{y},\theta\mathbf{b}) \in \mathbf{P}(\mathbf{x}). \tag{4}$$

This assumption implies that any proportional contraction of the desirable and the undesirable outputs is also feasible if the original combination of the desirable and the undesirable outputs is in the production technology set, for a given inputs $\mathbf{x}$. It also implies that the undesirable outputs are costly to be disposed of and that abatement activities would typically divert resources away from the production of desirable outputs and thus lead to lower desirable outputs with given inputs. The cost of abatement inevitably results in less production of the desirable outputs.

Another assumption related to the strong disposability of the desirable outputs is required, as follows:

$$\text{if } (\mathbf{y},\mathbf{b}) \in \mathbf{P}(\mathbf{x}) \text{ and } \mathbf{y}' \le \mathbf{y} \text{ then } (\mathbf{y}',\mathbf{b}) \in \mathbf{P}(\mathbf{x}). \tag{5}$$

This implies that if an observed outputs vector is feasible, then any outputs vector smaller than that is also feasible. It also means that some of the desirable outputs can always be disposed of without any cost.

The production technology set can be easily elaborated by employing the directional distance function. In other words, ease of computation as well as interpretation of the results are achieved if the production technology set is represented by the directional distance function. Maintaining the aforementioned assumptions, let $\mathbf{g} = (\mathbf{g}_\mathbf{y}, \mathbf{g}_\mathbf{b})$ be a direction vector, with $\mathbf{g} \in R_+^M \times R_+^J$. Then, the directional distance function is defined as follows:

$$\vec{D}_o(\mathbf{x},\mathbf{y},\mathbf{b};\mathbf{g}_\mathbf{y},\mathbf{g}_\mathbf{b}) = \max\{\beta : (\mathbf{y}+\beta\mathbf{g}_\mathbf{y}, \mathbf{b}-\beta\mathbf{g}_\mathbf{b} \in \mathbf{P}(\mathbf{x})\}. \tag{6}$$

This function seeks to increase the desirable outputs while simultaneously reducing the undesirable outputs. The direction vector, $\mathbf{g}$, determines the direction in which the desirable outputs increase and the undesirable outputs decrease. Following [4] and [12], the direction vector used in this study is $\mathbf{g}=(\mathbf{y},\mathbf{b})$. The production technology set and the directional distance function are depicted in Figure 1, in which it is assumed that DMU is producing good and bad outputs at point F. The production technology set is the inner area of the solid curve and the direction of the directional distance function of the DMU is depicted by the arrow.

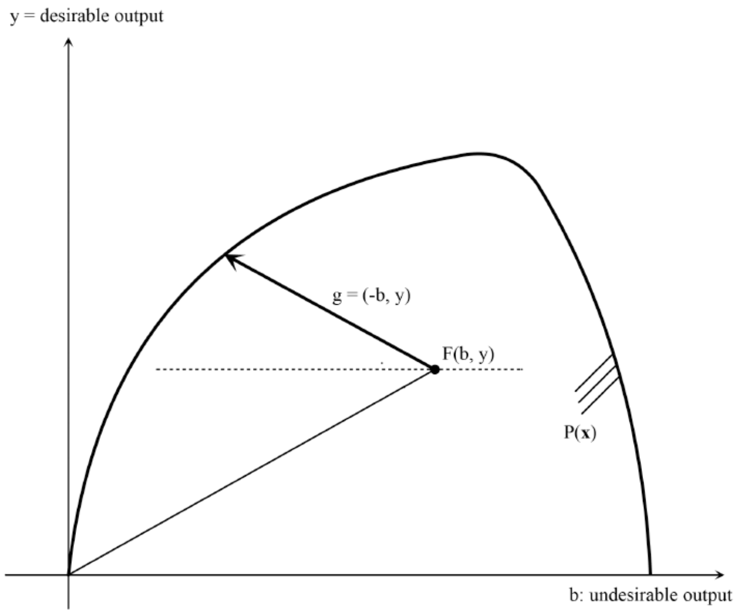


Figure 1. Directional output distance function with desirable and undesirable outputs.

To define and decompose the GML index, two definitions of the technology sets are essential for the calculation of the distance functions: a contemporaneous production technology set and a global production technology set. The contemporaneous production technology set is defined as $\mathbf{P}^t(\mathbf{x}^t)=\{(\mathbf{y}^t,\mathbf{b}^t)\,|\,\mathbf{x}^t \text{ can produce } (\mathbf{y}^t,\mathbf{b}^t)\}$ with $t=1,\cdots,T$. The contemporaneous production technology constructs a reference production set at each point in time t from the observations made at that time only [17]. The global production technology set is defined as a union of all contemporaneous technology set, i.

e., $\mathbf{P}^G(\mathbf{x}) = P^1(\mathbf{x}^1) \cup P^1(\mathbf{x}^1) \cup \cdots \cup P^T(\mathbf{x}^T)$. In this setup, the global production technology establishes a single reference production set from the observations throughout the whole set of observations and the entire time periods.

A contemporaneous Malmquist-Luenberger productivity index is defined on $\mathbf{P}^s(\mathbf{x}),\ s = t, t+1$ as:

$$ML^s = \left[\frac{(1+\vec{D}_o^s(\mathbf{x}^t, \mathbf{y}^t, \mathbf{b}^t; \mathbf{y}^t, \mathbf{b}^t))}{(1+\vec{D}_o^s(\mathbf{x}^{t+1}, \mathbf{y}^{t+1}, \mathbf{b}^{t+1}; \mathbf{y}^{t+1}, \mathbf{b}^{t+1}))} \right], \tag{7}$$

where the directional distance functions $\vec{D}_o^s(\mathbf{x}, \mathbf{y}, \mathbf{b}; \mathbf{y}, \mathbf{b}) = \max\{\beta : (\mathbf{y} + \beta\mathbf{y}, \mathbf{b} - \beta\mathbf{b} \in \mathbf{P}^s(\mathbf{x})\},\ s = t, t+1$, are defined on each of the contemporaneous technology set. Since $ML^t \neq ML^{t+1}$ without any restrictions on the two production technologies, the contemporaneous Malmquist-Luenberger productivity index is typically defined with a geometric mean form of two-period Malmquist-Luenberger productivity indexes as follows [4]:

$$\begin{aligned} &ML^{t,t+1} \\ &= \left[\frac{(1+\vec{D}_o^t(\mathbf{x}^t, \mathbf{y}^t, \mathbf{b}^t; \mathbf{y}^t, \mathbf{b}^t))}{(1+\vec{D}_o^t(\mathbf{x}^{t+1}, \mathbf{y}^{t+1}, \mathbf{b}^{t+1}; \mathbf{y}^{t+1}, \mathbf{b}^{t+1}))} \frac{(1+\vec{D}_o^{t+1}(\mathbf{x}^t, \mathbf{y}^t, \mathbf{b}^t; \mathbf{y}^t, \mathbf{b}^t))}{(1+\vec{D}_o^{t+1}(x^{t+1}, \mathbf{y}^{t+1}, \mathbf{b}^{t+1}; \mathbf{y}^{t+1}, \mathbf{b}^{t+1}))} \right]^{1/2}. \end{aligned} \tag{8}$$

This geometric mean form Malmquist-Luenberger productivity index is conventionally used as a measuring tool for productivity growth and its decomposed sources when the undesirable outputs are produced. Note that this productivity index, however, does not satisfy the circularity condition, proof of which can be found in Appendix A.1. Moreover, it is not free from the LP infeasibility in calculating the cross-period directional distance functions.

The geometric mean form of the conventional Malmquist-Luenberger productivity index can be decomposed as follows:

$$
\begin{aligned}
&ML^{t,t+1} \\
&= \frac{1+\vec{D}_o^t(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t;\mathbf{y}^t,\mathbf{b}^t)}{1+\vec{D}_o^{t+1}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1};\mathbf{y}^{t+1},\mathbf{b}^{t+1})} \\
&\times\left[\frac{(1+\vec{D}_o^{t+1}(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t;\mathbf{y}^t,\mathbf{b}^t))}{(1+\vec{D}_o^{t}(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t;\mathbf{y}^t,\mathbf{b}^t))}\frac{(1+\vec{D}_o^{t+1}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1};\mathbf{y}^{t+1},\mathbf{b}^{t+1}))}{(1+\vec{D}_o^{t}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1};\mathbf{y}^{t+1},\mathbf{b}^{t+1}))}\right]^{1/2} \\
&= MLEC^{t,t+1}\times MLTC^{t,t+1},
\end{aligned}
\tag{9}
$$

where $MLEC^{t,t+1}$ is the efficiency change and $MLTC^{t,t+1}$ is the technical change[2].

A global Malmquist-Luenberger productivity index, proposed in this study, is defined on the global production technology, $\mathbf{P}^G(\mathbf{x})$, as:

$$
GML^{t,t+1} = \frac{1+\vec{D}_o^G(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t;\mathbf{y}^t,\mathbf{b}^t)}{1+\vec{D}_o^G(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1};\mathbf{y}^{t+1},\mathbf{b}^{t+1})}, \tag{10}
$$

where the directional distance functions $\vec{D}_o^G(\mathbf{x}^s,\mathbf{y}^s,\mathbf{b}^s;\mathbf{y}^s,\mathbf{b}^s) = \max\{\beta : (\mathbf{y}^s+\beta\mathbf{y}^s, \mathbf{b}-\beta\mathbf{b}^s \in \mathbf{P}^G(\mathbf{x}^s)\}, s = t, t+1$ are defined on the global technology set.

Although both of the indexes compare $(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1})$ to $(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t)$, they use different production technology sets. The former uses two contemporaneous production technology sets and the latter uses one global production technology set. Since there is only one global production technology set over all time periods, it is not necessary to impose the geometric mean convention to the GML index[3].

$GML^{t,t+1}$ can be decomposed as follows:

[2] Note that further decomposition can be conducted. By incorporating a variable-returns-to-scale assumption in the contemporaneous technology set, for example,productivity growth can be decomposed into pure efficiency change, scale change, and technical change.

[3] The reason for imposing the geometric mean form to the contemporaneous ML index is to avoid choosing an arbitrary contemporaneous technology set [8,2].

$$
\begin{aligned}
& GML^{t,t+1} \\
&= \frac{1+\vec{D}_o^G(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t;\mathbf{y}^t,\mathbf{b}^t)}{1+\vec{D}_o^G(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1};\mathbf{y}^{t+1},\mathbf{b}^{t+1})} \\
&= \frac{1+\vec{D}_o^t(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t;\mathbf{y}^t,\mathbf{b}^t)}{1+\vec{D}_o^{t+1}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1};\mathbf{y}^{t+1},\mathbf{b}^{t+1})} \\
&\times \frac{(1+\vec{D}_o^G(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t;\mathbf{y}^t,\mathbf{b}^t))/(1+\vec{D}_o^t(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t;\mathbf{y}^t,\mathbf{b}^t))}{(1+\vec{D}_o^G(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1};\mathbf{y}^{t+1},\mathbf{b}^{t+1}))/(1+\vec{D}_o^{t+1}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1};\mathbf{y}^{t+1},\mathbf{b}^{t+1}))} \\
&= \frac{TE^t(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t;\mathbf{y}^t,\mathbf{b}^t)}{TE^{t+1}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1};\mathbf{y}^{t+1},\mathbf{b}^t)} \times \frac{BPG^{G,t}(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t;\mathbf{y}^t,\mathbf{b}^t)}{BPG^{G,t+1}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1};\mathbf{y}^{t+1},\mathbf{b}^{t+1})} \\
&= GMLEC^{t,t+1} \times GMLTC^{t,t+1}.
\end{aligned}
\tag{11}
$$

TE^s in equation (11) represents the efficiency component at time period s, while $BPG^{G,s}$ in the second term of the equation is the best practice gap between $\mathbf{P}^G$ and $\mathbf{P}^s$ along direction of $(\mathbf{y}^s, \mathbf{b}^s)$. Changes in efficiency are measured by $GMLEC^{t,t+1}$, which represents a movement of countries towards the best practice frontier. If $GMLEC^{t,t+1} > 1$, then there has been a movement toward the frontier in period $t+1$, and hence becomes more efficient. If $GMLEC^{t,t+1} < 1$, then it signifies that the country is farther away from the frontier in $t+1$ than in t, and hence becomes less efficient. $GMLTC^{t,t+1}$ is the change in BPG, and provides a new measure of technical change. $GMLTC^{t,t+1} > (<) 1$ indicates that the production technology in period $t+1$ is closer to (or, farther away from) the global production technology than is the production technology in period t. Hence, $GMLTC^{t,t+1} > 1$ signifies technical progress between t and $t+1$, and $GMLTC^{t,t+1} < 1$ signifies technical regress. If there have been no changes in inputs and outputs over the two periods, then $GML^{t,t+1} = 1$. If there has been an increase in productivity then $GML^{t,t+1} > 1$, and finally, a decrease when $GML^{t,t+1} < 1$. Unlike the conventional geometric mean form of the ML index, the global Malmquist-Luenberger productivity index is circular. The proof of the circularity of the GML index is given in Appendix A.2.

The directional distance function can be calculated in several ways. [11] and [7] specify the output distance function in a translog form, and [10], [12], [13] and [4] utilized the linear-programming approach for evaluating the directional distance functions. In this study, the linear-programming technique is employed to determine the directional distance functions. Following [4], a data envelopment analysis (DEA) model can be used in constructing the contemporaneous and global production technology set based on the aforementioned assumptions of the production technology set. Let us assume that there are $k = 1, \cdots, K$ producers of inputs and outputs $(\mathbf{x}_k^t, \mathbf{y}_k^t, \mathbf{b}_k^t)$ for time period $t = 1, \cdots, T$. Using this data in the DEA framework, an output set, $\mathbf{P}^s(\mathbf{x}^t)$, can be established as follows:

$$
\begin{aligned}
\mathbf{P}^s(\mathbf{x}^t) = \{(\mathbf{y}^t, \mathbf{b}^t) \mid & \sum_{k=1}^{K} z_k^s y_{km}^s \geq y_m^t, \quad m = 1, \cdots, M \\
& \sum_{k=1}^{K} z_k^s b_{kj}^s = b_j^t, \quad j = 1, \cdots, J \\
& \sum_{k=1}^{K} z_k^s x_{kn}^s \leq x_n^t, \quad n = 1, \cdots, N \\
& z_k^t \geq 0, \quad k = 1, \cdots, K\},
\end{aligned}
\tag{12}
$$

where z_k^t are the intensity variables to each observation in constructing the production possibility frontier.

To calculate and decompose the GML index of producer k' between t and $t+1$, we need to solve four different linear-programming problems. Two use observations and technology for time period t, or $t+1$; two use observations for time period t, or $t+1$ and technology for all time periods: $\vec{D}_o^t(\mathbf{x}^t, \mathbf{y}^t, \mathbf{b}^t; \mathbf{y}^t, \mathbf{b}^t)$, $\vec{D}_o^{t+1}(\mathbf{x}^{t+1}, \mathbf{y}^{t+1}, \mathbf{b}^{t+1}; \mathbf{y}^{t+1}, \mathbf{b}^{t+1})$, $\vec{D}_o^G(\mathbf{x}^t, \mathbf{y}^t, \mathbf{b}^t; \mathbf{y}^t, \mathbf{b}^t)$, and $\vec{D}_o^G(\mathbf{x}^{t+1}, \mathbf{y}^{t+1}, \mathbf{b}^{t+1}; \mathbf{y}^{t+1}, \mathbf{b}^{t+1})$. By utilizing the empirical output set shown in equation (1), each contemporaneous directional distance function at $s = t, t+1$ can be calculated by solving the following LP problems [4]:

$$
\begin{aligned}
&\vec{D}_o^s(\mathbf{x}^s,\mathbf{y}^s,\mathbf{b}^s;\mathbf{y}^s,-\mathbf{b}^s)=\max\beta \\
&s.t\quad \sum_{k=1}^{K} z_k^s y_{km}^s \geq (1+\beta) y_m^s,\ \ m=1,\cdots,M \\
&\qquad \sum_{k=1}^{K} z_k^s b_{kj}^s = (1-\beta) b_j^s,\quad j=1,\cdots,J \\
&\qquad \sum_{k=1}^{K} z_k^s x_{kn}^s \leq x_n^s,\qquad n=1,\cdots,N \\
&\qquad z_k^s \geq 0,\qquad k=1,\cdots,K,
\end{aligned}
\tag{13}
$$

where z_k^s are the intensity variables.

In contrast to the above LP equation, the global directional distance function exploits all observations over all time periods in constructing the production possibility set. The global directional distance functions are computed as follows:

$$
\begin{aligned}
&\vec{D}_o^G(\mathbf{x}^s,\mathbf{y}^s,\mathbf{b}^s;\mathbf{y}^s,-\mathbf{b}^s)=\max\beta \\
&s.t\quad \sum_{t=1}^{T}\sum_{k=1}^{K} z_k^t y_{km}^t \geq (1+\beta) y_m^s,\ \ m=1,\cdots,M \\
&\qquad \sum_{t=1}^{T}\sum_{k=1}^{K} z_k^t b_{kj}^t = (1-\beta) b_j^s,\quad j=1,\cdots,J \\
&\qquad \sum_{t=1}^{T}\sum_{k=1}^{K} z_k^t x_{kn}^t \leq x_n^s,\qquad n=1,\cdots,N \\
&\qquad z_k^t \geq 0,\qquad k=1,\cdots,K.
\end{aligned}
\tag{14}
$$

The optimal solutions of equation (13) and (14) are employed in the calculation and decomposition of the GML index.

3. Data and Empirical Study

We obtain the data on six variables, namely, GDP, CO_2 SO_x, labor force, capital stock, and commercial energy consumption, for 28 OECD countries over the period 1990-2004[4]. Among the first three variables, GDP is chosen as a proxy of the desirable output, whereas CO_2 and SO_x are chosen as proxies of undesirable outputs. Labor force, capital stock, and commercial energy consumption are the inputs of the production technology. Data on GDP, labor force, and capital stock are collected from the merged data set from the Penn World Table (Mark 5.6) and the Penn World Table (Mark 6.2). Since the capital stock over the period 1990-2004 is not available for all countries, we estimated the capital stock using investment series contained in the Penn World Table (Mark 6.2) based on the capital stock stated in the Penn World Table (Mark 5.6) employing perpetual inventory method in which the depreciation rate is assumed to be 10% per year following [3]. GDP and capital stock are transformed to be measured in 2000 US dollars. Data on CO_2 and SO_x are taken from OECD Environmental Data [15] and energy consumption data are taken from the website of World Bank's World Development Indicator.

[4]Data on the Republic of Czech and the Republic of Slovak do not exist over the period 1990-1995. Therefore, these two countries are excluded from the empirical analysis.

Table 1. Descriptive statistics of variables used in this study: Average and standard deviation of growth rate (1990–2004)

Country	GDP		CO_2		SOx		Energy		Capital		Labor	
	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.
Australia	0.035	0.012	0.023	0.018	0.036	0.086	0.020	0.023	0.072	0.038	0.014	0.002
Austria	0.021	0.010	0.020	0.044	−0.067	0.076	0.021	0.033	0.054	0.029	0.005	0.005
Belgium	0.019	0.013	0.005	0.038	−0.056	0.060	0.012	0.030	0.053	0.035	0.005	0.003
Canada	0.030	0.023	0.018	0.020	−0.019	0.081	0.018	0.017	0.059	0.048	0.012	0.001
Denmark	0.022	0.016	0.005	0.110	−0.098	0.241	0.010	0.057	0.042	0.033	0.001	0.002
Finland	0.020	0.043	0.017	0.077	−0.070	0.107	0.020	0.041	0.029	0.014	0.002	0.003
France	0.019	0.015	0.007	0.037	−0.067	0.068	0.014	0.029	0.059	0.026	0.008	0.004
Germany	0.016	0.014	−0.009	0.022	−0.141	0.092	−0.002	0.017	0.043	0.029	0.002	0.003
Greece	0.030	0.018	0.021	0.026	0.012	0.044	0.023	0.017	0.042	0.023	0.012	0.005
Hungary	0.021	0.043	−0.015	0.033	−0.093	0.076	−0.005	0.033	−0.011	0.030	0.002	0.004
Ireland	0.068	0.032	0.022	0.033	−0.059	0.101	0.028	0.031	0.099	0.046	0.020	0.005
Iceland	0.028	0.032	0.000	0.000	0.004	0.074	0.035	0.043	0.079	0.051	0.011	0.002
Italy	0.014	0.011	0.009	0.020	−0.087	0.037	0.016	0.026	0.056	0.036	0.004	0.003
Japan	0.012	0.015	0.009	0.019	−0.017	0.028	0.013	0.020	0.052	0.036	0.006	0.005
Korea, Rep.	0.057	0.046	0.054	0.064	−0.041	0.059	0.063	0.053	0.108	0.075	0.019	0.006
Luxembourg	0.048	0.030	0.005	0.097	−0.093	0.174	0.022	0.054	0.076	0.038	0.011	0.002
Mexico	0.029	0.032	0.018	0.031	0.007	0.055	0.021	0.020	0.069	0.034	0.027	0.003
Netherlands	0.023	0.015	0.036	0.050	−0.073	0.054	0.015	0.021	0.056	0.032	0.006	0.003
Norway	0.033	0.016	0.017	0.063	−0.049	0.062	0.019	0.040	0.033	0.020	0.008	0.002
New Zealand	0.032	0.019	0.012	0.025	0.027	0.062	0.018	0.025	0.070	0.035	0.014	0.003
Poland	0.037	0.022	−0.011	0.031	−0.064	0.044	−0.006	0.033	0.040	0.048	0.006	0.002
Portugal	0.021	0.018	0.031	0.062	−0.020	0.141	0.030	0.035	0.069	0.045	0.006	0.001
Spain	0.028	0.016	0.035	0.043	−0.033	0.078	0.033	0.028	0.082	0.034	0.012	0.002
Sweden	0.020	0.023	0.003	0.054	−0.066	0.058	0.010	0.036	0.030	0.010	0.004	0.002
Switzerland	0.009	0.014	−0.017	0.048	−0.131	0.124	0.006	0.022	0.021	0.009	0.005	0.002
Turkey	0.034	0.050	0.037	0.055	0.013	0.058	0.033	0.049	0.075	0.041	0.024	0.003

U.K.	0.025	0.015	−0.002	0.024	−0.098	0.070	0.007	0.018	0.071	0.018	0.004	0.000
U.S.A.	0.031	0.016	0.013	0.014	−0.031	0.040	0.014	0.013	0.079	0.020	0.012	0.001
Total	0.028	0.027	0.013	0.049	−0.049	0.100	0.018	0.035	0.057	0.042	0.009	0.007

We begin with a summary of the average annual growth rate of desirable and undesirable outputs, capital stock, labor, and energy consumption for each country. Summary statistics of the variables used in this study are shown in Table 1. Average growth in GDP is 2.8% per year over the entire 1990–2004 period for our sample. Ireland, the Republic of Korea, and Luxembourg had the highest average annual growth in GDP (6.8%, 5.7%, and 4.8%, respectively), and Switzerland, Japan, and Italy had the lowest (less than 1.5%.) Average growth rates of CO_2 emission and SO_x emission are 1.3% and –4.9 %, respectively. During the study period, only five countries, i.e., Switzerland, Hungary, Poland, Germany, and the UK had decreased CO_2 emissions, while most countries had increased emissions. In contrast to CO_2 emissions, most countries had decreased their SO_x emissions, while those of six countries, i.e., Australia, New Zealand, Turkey, Greece, Mexico, and Iceland had increased the emissions. The average annual growth rate of energy consumption for OECD countries is 1.8%. The Republic of Korea had the highest rank in terms of energy consumption, with a growth rate more than double that of other countries. Energy consumption in Poland, Hungary, and Germany decreased over the study period, while other countries increased their use of energy. The capital stock and the labor force of all countries increased except for the capital stock of Hungary. The annual growth rates of the capital stock and labor force of OECD countries are 5.7% and 0.7% per year, respectively. During the study period, the highest accumulation rate of capital stock was in the Republic of Korea, Ireland, and Spain. Although the labor force of all OECD countries increased, Mexico and Turkey experienced the fastest growth in the labor force.

To examine the emissions of by-product of OECD countries on the whole, trends in CO_2 and SO_x emissions are depicted in Figure 2. CO_2 emission continuously increased over the study period except for during 2000-2001, while SO_x emission constantly decreased. Although a consistent increasing trend in CO_2 emissions cannot be found, it can be speculated that the annual growth rate of emissions ranges from -0.3% to 3.3% [5]. The annual rate of reduction in SO_x emission varies from $0.6\,\%$ to $0.9\,\%$. From 1994 to 1995, SO_x emissions dramatically decreased. However, it seems that the decreasing rate of SO_x emissions rarely changed after 1995. Consistent increases in energy consumption except for during 2000-2001 was also found. The correlation between energy consumption and CO_2 emissions is 0.994. It appears that the increase in CO_2 emissions has been caused by high energy consumption.

[5] The negative growth in CO_2 emissions occurred only over the period 2000-2001.

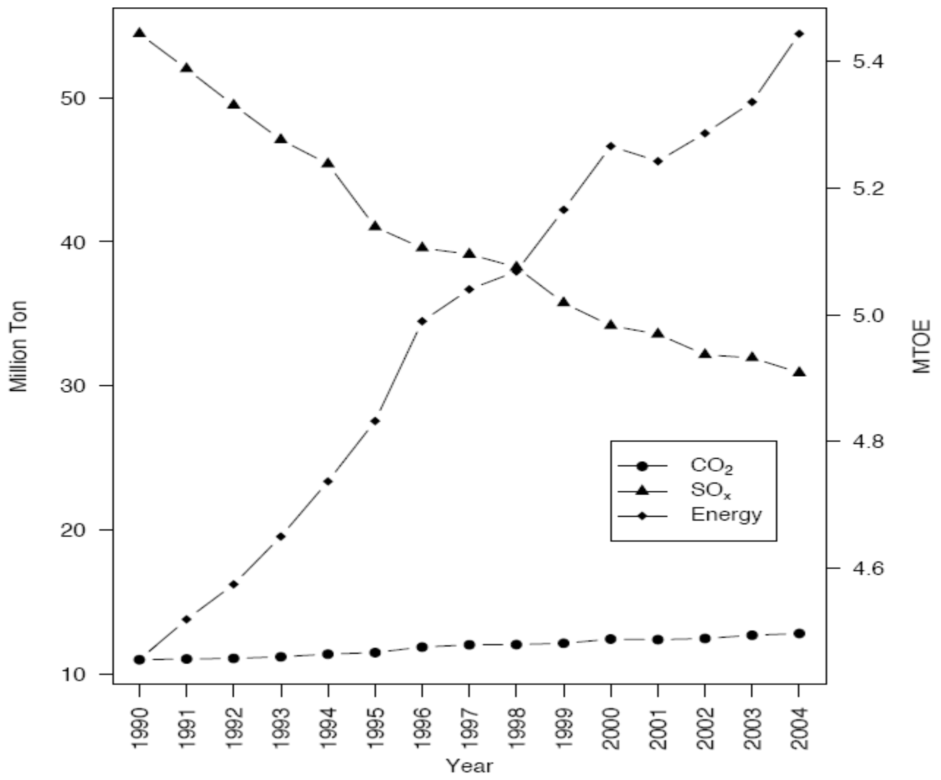


Figure 2. CO_2 **and** SO_x **emissions, and energy consumption of OECD countries**

Table 2. Productivity change, efficiency change, technical change of OECD countries over 1990–2004: Comparison of proposed methodology and the conventional measures of [4]

Country	GML index			ML index			Differences (%)		
	GML	GMLEC	GMLTC	ML	MLEC	MLTC	(1)–(4)	(2)–(5)	(3)–(6)
	(1)	(2)	(3)	(4)	(5)	(6)			
Australia	0.9985	1.0108	0.9878	0.9934	1.0108	0.9827	0.51%	0.00%	0.51%
Austria	1.0000	1.0000	1.0000	0.9997	1.0000	0.9997	0.03%	0.00%	0.03%
Belgium	0.9915	0.9928 *	0.9986	0.9956	0.9928 *	1.0027	-0.41%	0.00%	-0.41%
Canada	0.9919	0.9972	0.9947	0.9850	0.9972	0.9877	0.69%	0.00%	0.70%
Denmark	1.0100	1.0088	1.0012	1.0031	1.0088	0.9944	0.69%	0.00%	0.68%
Finland	1.0023	1.0016	1.0007	1.0002	1.0016	0.9987	0.21%	0.00%	0.20%
France	0.9940	0.9954 **	0.9986	0.9978	0.9954 **	1.0025	-0.38%	0.00%	-0.39%

Germany	1.0028	1.0020	1.0008	1.0028	1.0020	1.0008	0.00%	0.00%	0.00%
Greece	1.0055	1.0184 *	0.9873	0.9940	1.0184 *	0.9761 **	1.15%	0.00%	1.12%
Hungary	0.9823	0.9890	0.9932	0.9600 **	0.9890	0.9706 **	2.23%	0.00%	2.26%
Ireland	1.0075	1.0075	1.0000	0.9902	1.0075	0.9829	1.73%	0.00%	1.71%
Iceland	0.9887	0.9982	0.9905	0.9859	0.9982	0.9876 ***	0.28%	0.00%	0.29%
Italy	0.9972	0.9985	0.9986	0.9994	0.9985	1.0009	-0.22%	0.00%	-0.23%
Japan	0.9921	0.9890	1.0031	1.0062	0.9890	1.0173 **	-1.41%	0.00%	-1.42%
Korea, Rep.	0.9872	0.9892	0.9980	0.9788 *	0.9892	0.9895	0.84%	0.00%	0.85%
Luxembourg	1.0000	1.0000	1.0000	1.0015	1.0000	1.0015	-0.15%	0.00%	-0.15%
Mexico	0.9835 **	1.0017	0.9818 **	0.9898 *	1.0017	0.9881 **	-0.63%	0.00%	-0.63%
Netherlands	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.00%	0.00%	0.00%
Norway	1.0134 **	1.0005	1.0129 **	1.0100 ***	1.0005	1.0096 **	0.34%	0.00%	0.33%
NewZealand	1.0000	1.0000	1.0000	0.9973	1.0000	0.9973	0.27%	0.00%	0.27%
Poland	0.9796	1.0000	0.9796	0.9389 ***	1.0000	0.9389 ***	4.07%	0.00%	4.07%
Portugal	0.9940	1.0000	0.9940	0.9952 **	1.0000	0.9952 **	-0.12%	0.00%	-0.12%
Spain	0.9921	0.9986	0.9935	0.9931 **	0.9986	0.9946 ***	-0.10%	0.00%	-0.11%
Sweden	1.0082	1.0064	1.0018	1.0065	1.0064	1.0001	0.17%	0.00%	0.17%
Switzerland	1.0000	1.0000	1.0000	0.9998	1.0000	0.9998	0.02%	0.00%	0.02%
Turkey	0.9818 **	1.0000	0.9818 **	0.9954 **	1.0000	0.9954 **	-1.36%	0.00%	-1.36%
U.K.	0.9964	0.9979	0.9985	0.9976	0.9979	0.9997	-0.12%	0.00%	-0.12%
U.S.A.	0.9899 ***	0.9945	0.9953 **	0.9916 ***	0.9945	0.9970 **	-0.17%	0.00%	-0.17%
Total	0.9960	0.9999	0.9961	0.9931	0.9999	0.9932	0.29%	0.00%	0.29%

Note: *, **, and *** denote an estimate significantly different from 1 at the 10%, 5%, and 1% level. Wilcoxon test was performed to test the null hypothesis.

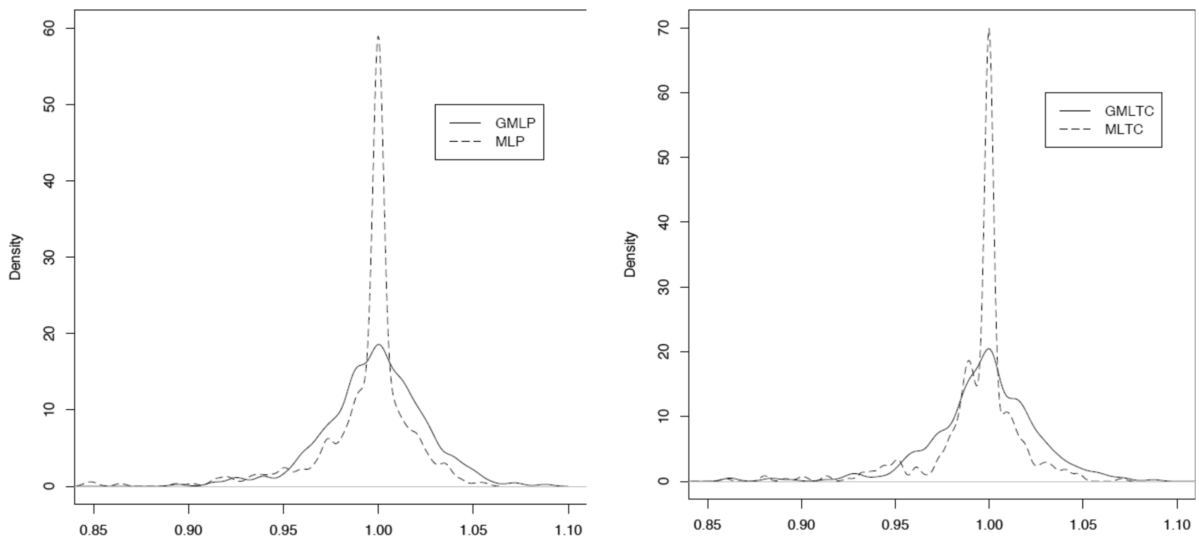
The suggested approach outlined in Section 2 constructs a global best-practice frontier from the data set, whilst the conventional Malmquist-Luenberger productivity index establishes a time-specific best-practice frontier. The empirical results are summarized in Table 2, which presents the results of applying our methodology as well as that of [4] to the data set[6]. Differences between values of our methodology and those of [4] are also presented in the last three columns of the Table 2. Country-specific productivity growth, efficiency change, and technical change are presented in each row of the table and their overall average change is shown in the bottom row. Recall that index values greater (less) than unity signify improvements (deterioration) in the relevant performance.

The average annual productivity change in the GML index was $-0.4\,\%$. This average TFP measure was the product of a negative efficiency change of less than $0.1\,\%$ and a negative technical change of $0.4\,\%$. In our sample countries, Norway experienced the highest growth in TFP and Poland experienced the highest decline in the index regardless of methodologies. Norway is one of the countries that experienced positive efficiency change as well as positive technical change. It also ranks as the most productive country with the highest growth in technical change, which is much higher than those of other countries. Denmark, Sweden, Germany, and Finland also experienced positive productivity growth in both methodologies. Note that these countries, except for Germany, are located in the Northern Europe. Ireland and Greece respectively rank the fourth and fifth with positive productivity growth indexes in our methodology, but application of the geometric mean form of the Malmquist-Luenberger productivity index produces negative productivity growth indexes. On the other hand, Japan and Luxembourg respectively rank the third and sixth in terms of productivity growth using the geometric mean form index, but they had negative productivity growth in our methodology.

In addition to the country-specific performances, a comparison of the results between the two methodologies needs to be made. As can be seen in equations (9) and (11), the two

[6] In calculating the conventional Malmquist-Luenberger productivity index, LP infeasibility occurred. To avoid this infeasibility problem, we employed window analysis.

indexes of efficiency change are identical between our methodology and that of [4], while the technical change indexes are different. Therefore, the difference between $GML^{t,t+1}$ and $ML^{t,t+1}$ mainly comes from the difference between $GMLTC^{t,t+1}$ and $MLTC^{t,t+1}$. Differences in the productivity growth indexes as well as the technical change indexes between the two methodologies can be found in Figure 3. Empirical density functions of $GML^{t,t+1}$ and $ML^{t,t+1}$ can be found in the left panel of Figure 3, which shows that productivity growth estimated from GML index are more dispersed than that those from ML. This less dispersed ML inevitably arises because it utilized the geometric mean form of two adjacent productivity growth indexes. Similar patterns can be found in the technical change indexes.



(a) Productivity growth index (b) Technical change index

Figure 3. Kernel density of performance index: comparison between proposed methodology and conventional approach

A Wilcoxon rank sum test was performed to test the null hypothesis that the two productivity measures and their components are the same. The test statistics and p-values are listed in each row of Table 3 with respect to each hypothesis. The TFP index values were found not to differ between the methodologies. The efficiency change indexes do not differ between the methodologies. However, the technical change indexes are different between the two methodologies, which signifies that the main difference between the two TFP measures is the difference in the technical change indexes. In other words, the main factor of productivity growth is technical change.

3. Hypothesis test using the Wilcoxon-test

Null hypothesis	Statistics	p-value	Result
GML = ML	80907.50	0.1981	Fail to reject
GMLEC = MLEC	76832.00	1.0000	Fail to reject
GMLTC = MLEC	82299.50	0.0842	Rejected

To empirically test whether the transitivity is satisfied in the GML index and the conventional ML index, the productivity and its sources for Norway are listed in Table 4 as an example. The first three columns report GML index and its decomposed sources, and the final three columns report ML and its decompositions. Rows correspond to time periods. Cumulative productivity in 2004 is 20.4% greater than in 1990. GML index calculated using 1990 and 2004 data generates the same value, verifying that it is circular. The efficiency change component of GML index is also circular, and results in a 0.6% improvement. The technical change of GML index, GMLTC, is also circular, and increased by 19.6%. Turning to the conventional Malmquist-Luenberger index, reported in the final three columns, the productivity index and the technical change index are not circular, which should be interpreted with caution.

4. Global Malmquist-Luenberger index and conventional Malmquist-Luenberger index of Norway

	GML index			ML index		
	GML	GMLEC	GMLTC	ML	MLEC	MLTC
1990-1991	1.0233	1.0066	1.0167	1.0204	1.0066	1.0137
1991-1992	1.0126	1.0000	1.0126	1.0082	1.0000	1.0082
1992-1993	1.0191	1.0000	1.0191	1.0043	1.0000	1.0043
1993-1994	1.0368	1.0000	1.0368	1.0129	1.0000	1.0129
1994-1995	1.0125	1.0000	1.0125	1.0094	1.0000	1.0094
1995-1996	1.0295	1.0000	1.0295	1.0051	1.0000	1.0051
1996-1997	1.0073	1.0000	1.0073	1.0026	1.0000	1.0026
1997-1998	0.9943	0.9902	1.0041	1.0055	0.9902	1.0154
1998-1999	0.9843	0.9864	0.9978	1.0103	0.9864	1.0242
1999-2000	1.0514	0.9909	1.0611	1.0393	0.9909	1.0488
2000-2001	1.0089	1.0228	0.9864	1.0126	1.0228	0.9900
2001-2002	1.0307	1.0101	1.0204	1.0163	1.0101	1.0061
2002-2003	0.9800	1.0000	0.9800	0.9939	1.0000	0.9939
2003-2004	0.9988	1.0000	0.9988	1.0005	1.0000	1.0005
Cum. prod.	1.2040	1.0066	1.1962	1.1501	1.0066	1.1427
1990-2004	1.2040	1.0066	1.1962	1.0852	1.0066	1.0781

4. Conclusion

As the ecological concerns have increased in recent decades, measures of economic growth incorporating the negative effects of environmentally bad outputs have long been needed to be developed to enable the estimation of an unbiased productivity growth. Although the Malmquist-Luenberger productivity index has been recognized as a practical tool for measuring unbiased measurement, it is not circular and it poses potential LP infeasibility problems. To avoid these problems, we developed an alternative measure, namely the global Malmquist-Luenberger productivity index. This alternative measure is circular as well as free from LP infeasibility.

The methodology is applied to the 28 OECD countries over the period 1990-2004. To compare our results with previous studies, the conventional geometric mean form of two adjacent Malmquist-Luenberger productivity indexes was also calculated. The test statistics, which test the null hypothesis that the two productivity measures and their components are the same, show that (i) we faild to reject the null hypothesis that the two productivity indexes are the same and (ii) it is rejected that the two technical change indexes are the same. The efficiency change components of the two methodologies are identical. Most of the Northern European countries rank high in terms of the productivity growth.

The global Malmquist-Luenberger productivity index should be recomputed if cross-sectional data for a new period is added to the data set, since the global technology might change from the addition of the data. As [5] asserted, however, economic history has to be rewritten when new data are incorporated. The global Malmquist-Luenberger productivity index could be employed in rewriting economic history, whilst it is hard to be discovered with the conventional Malmquist-Luenberger productivity index. This revision of history could be quantitative rather than qualitative if the proposed methodology is employed in estimating productivity growth incorporating the negative effect of environmentally harmful outputs.

Appendix A

A.1. Proof of Non-transitivity of the contemporaneous Malmquist-Luenberger productivity index

The transitivity of index number is defined as follows:

$$I^{t,t+2} = I^{t,t+1} \times I^{t+1,t+2}. \tag{A–1}$$

where $I^{t,t+1}$, $I^{t+1,t+2}$, and $I^{t,t+2}$ represent the index between period t and $t+1$, $t+1$ and $t+2$, and t and $t+2$, respectively.

Geometric mean form of Malmquist-Luenberger productivity index between t and $t+1$, $t+1$ and $t+2$, and t and $t+2$ are respectively as follows:

$$ML^{t,t+1} = \left[\frac{1+\vec{D}_o^t(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t;\mathbf{y}^t,-\mathbf{b}^t)}{1+\vec{D}_o^t(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1};\mathbf{y}^{t+1},-\mathbf{b}^{t+1})} \cdot \frac{1+\vec{D}_o^{t+1}(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t;\mathbf{y}^t,-\mathbf{b}^t)}{1+\vec{D}_o^{t+1}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1};\mathbf{y}^{t+1},-\mathbf{b}^{t+1})} \right]^{1/2}. \tag{A–2}$$

$$ML^{t+1,t+2} = \left[\frac{1+\vec{D}_o^{t+1}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1};\mathbf{y}^{t+1},-\mathbf{b}^{t+1})}{1+\vec{D}_o^{t+1}(\mathbf{x}^{t+2},\mathbf{y}^{t+2},\mathbf{b}^{t+2};\mathbf{y}^{t+2},-\mathbf{b}^{t+2})} \cdot \frac{1+\vec{D}_o^{t+2}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1};\mathbf{y}^{t+1},-\mathbf{b}^{t+1})}{1+\vec{D}_o^{t+2}(\mathbf{x}^{t+2},\mathbf{y}^{t+2},\mathbf{b}^{t+2};\mathbf{y}^{t+2},-\mathbf{b}^{t+2})} \right]^{1/2}. \tag{A–3}$$

$$ML^{t,t+2} = \left[\frac{1+\vec{D}_o^t(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t;\mathbf{y}^t,-\mathbf{b}^t)}{1+\vec{D}_o^t(\mathbf{x}^{t+2},\mathbf{y}^{t+2},\mathbf{b}^{t+2};\mathbf{y}^{t+2},-\mathbf{b}^{t+2})} \cdot \frac{1+\vec{D}_o^{t+2}(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t;\mathbf{y}^t,-\mathbf{b}^t)}{1+\vec{D}_o^{t+2}(\mathbf{x}^{t+2},\mathbf{y}^{t+2},\mathbf{b}^{t+2};\mathbf{y}^{t+2},-\mathbf{b}^{t+2})} \right]^{1/2}. \tag{A–4}$$

Multiplication of both side of equation (A-2) and (A-3) is as follows:

$$
\begin{aligned}
&ML^{t,t+1} \times ML^{t+1,t+2} \\
&= \left[\frac{1+\vec{D}_o^{t}(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t;\mathbf{y}^t,-\mathbf{b}^t)}{1+\vec{D}_o^{t}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1};\mathbf{y}^{t+1},-\mathbf{b}^{t+1})} \times \frac{1+\vec{D}_o^{t+1}(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t;\mathbf{y}^t,-\mathbf{b}^t)}{1+\vec{D}_o^{t+1}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1};\mathbf{y}^{t+1},-\mathbf{b}^{t+1})} \right]^{1/2} \\
&\times \left[\frac{1+\vec{D}_o^{t+1}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1};\mathbf{y}^{t+1},-\mathbf{b}^{t+1})}{1+\vec{D}_o^{t+1}(\mathbf{x}^{t+2},\mathbf{y}^{t+2},\mathbf{b}^{t+2};\mathbf{y}^{t+2},-\mathbf{b}^{t+2})} \times \frac{1+\vec{D}_o^{t+2}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1};\mathbf{y}^{t+1},-\mathbf{b}^{t+1})}{1+\vec{D}_o^{t+2}(\mathbf{x}^{t+2},\mathbf{y}^{t+2},\mathbf{b}^{t+2};\mathbf{y}^{t+2},-\mathbf{b}^{t+2})} \right]^{1/2} \qquad \text{(A–5)} \\
&= [\frac{(1+\vec{D}_o^{t}(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t;\mathbf{y}^t,-\mathbf{b}^t))}{(1+\vec{D}_o^{t}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1};\mathbf{y}^{t+1},-\mathbf{b}^{t+1}))} \times \frac{(1+\vec{D}_o^{t+1}(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t;\mathbf{y}^t,-\mathbf{b}^t))}{(1+\vec{D}_o^{t+1}(\mathbf{x}^{t+2},\mathbf{y}^{t+2},\mathbf{b}^{t+2};\mathbf{y}^{t+2},-\mathbf{b}^{t+2}))} \\
&\times \frac{(1+\vec{D}_o^{t+2}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1};\mathbf{y}^{t+1},-\mathbf{b}^{t+1}))}{(1+\vec{D}_o^{t+2}(\mathbf{x}^{t+2},\mathbf{y}^{t+2},\mathbf{b}^{t+2};\mathbf{y}^{t+2},-\mathbf{b}^{t+2}))}]^{1/2}
\end{aligned}
$$

Then, $ML^{t,t+1} \times ML^{t+1,t+2} \neq ML^{t,t+2}$.

A.2. Proof of transitivity of the global Malmquist-Luenberger productivity index

$$
\begin{aligned}
&GML^{t,t+1} \times GML^{t+1,t+2} \\
&= \frac{1+\vec{D}_o^{G}(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t;\mathbf{y}^t,-\mathbf{b}^t)}{1+\vec{D}_o^{G}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1};\mathbf{y}^{t+1},-\mathbf{b}^{t+1})} \times \frac{1+\vec{D}_o^{G}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1};\mathbf{y}^{t+1},-\mathbf{b}^{t+1})}{1+\vec{D}_o^{G}(\mathbf{x}^{t+2},\mathbf{y}^{t+2},\mathbf{b}^{t+2};\mathbf{y}^{t+2},-\mathbf{b}^{t+2})} \\
&= \frac{1+\vec{D}_o^{G}(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t;\mathbf{y}^t,-\mathbf{b}^t)}{1+\vec{D}_o^{G}(\mathbf{x}^{t+2},\mathbf{y}^{t+2},\mathbf{b}^{t+2};\mathbf{y}^{t+2},-\mathbf{b}^{t+2})} \qquad \text{(B–1)} \\
&= GML^{t,t+2}
\end{aligned}
$$

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