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Productivity of and Returns to Knowledge Investments

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PRODUCTIVITY OF AND RETURNS TO KNOWLEDGE INVESTMENTS

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Abstract
Information and knowledge are essential to the decision making of firms. However, information is a primitive in the formation of knowledge. Information and the related concepts of risk and surprise are primarily of importance for rational decision making while knowledge is a form of (non-material) capital to be used as a resource in the transformation of different inputs into valuable outputs. Knowledge can be embodied in educated labor, in material capital, as patented production recipes or as contents of publically available documents and other media. In the paper optimality aspects of the acquisition of knowledge capital is analyzed. Estimates of private and public returns to investments in knowledge are reported. Some economic consequences of frictions over time and space are analyzed and new models containing such frictions are proposed.

Information and Data in the Financial Evaluation of Firms

Information in the form of statistical data is of importance in evaluation of firms in the financial markets. Information on profits, sales, new products and changes in the leadership of firms, traded in the stock market, often has immediate consequences for the evaluation among stock holders. Information on administration of businesses has always been considered important. However, the role of macro-economic conditions in the determination of prices of different securities was rarely discussed before the 1970s.

In the 1960s Sharpe (1963), Lintner (1965) and Mossin (1966) proposed the use of the Capital Asset Pricing Model (CAPM) as a way of predicting the returns on financial capital of firms, traded in the market for securities. According to this model the percentage returns of a given firm would be determined by a statistical constant, β_i times the returns on the capital market as a whole (often represented by the returns on an index portfolio). β_i equal to 1 would indicate that the individual share i would have the same risk as the capital market as a whole.

In the mid 1970s Stephen Ross proposed a generalization of the CAPM model of valuation of firms in the market for securities – the Arbitrage Pricing Theory (APT), Ross (1975). Ross´ idea was quite close to the basic ideas behind information theory. Ross made a distinction between information about the firm and macro-economic information. He presumed that the general development of demand, as represented by the growth rate of GDP, the rate of inflation, the development in export markets and similar macro-economic variables would influence different firms and the returns to their shares to different degrees. For each such variable a separate β_{ik} would be estimated. β_{ik} would then indicate the elasticity of the returns of firm i with respect to the *information* on macro-economic variable k. With arguments similar to information theory, information in the APT- model is defined as *surprises* of the development of the macro-economic variables. If, for example, the rate of growth of the GDP would be larger than expected, then there would be an impact on the pricing of securities traded in the stock market and thus on expected returns. However, stock market prices would be left uninfluenced if there would be no new information, (i.e. no surprises).

The distinction between information and knowledge can be highlighted in the context of the APT-models. The flow of information is important for pricing and returns, but only if analysts have substantial knowledge about the macro-economic factors constituting the explanatory variables of the APT-model. Knowledge is of use in the construction of the model and the modelling and formation of expectations, which are activities, exogenous to the APT-model.

The Optimal Use of Unskilled and Educated Labor in Firms

The most elementary issue in decision making on the use of knowledge in firms is in decision making on employment of labor. Any firm has to decide on the level of knowledge and skills, acquired by education, to be employed in the firm. For simplicity we can assume that there are only two kinds of labor available for employment. The first kind of labor has no education above the compulsory level, acquired in the young years of life, when no alternative employment has been possible. There are thus no education investment costs to be compensated for by the employer of such labor. With the employment of such labor there are costs per unit, proportional to the wage rate.

For the other, educated category of labor, the wage rate of uneducated labor plus returns to the cost of education capital must be paid. If the level of education of the second type of labor is given, the wage rate of the educated category of labor will be higher by some percentage. In the case of the current US data the high education group could on the average have a wage rate as large as three times the wage rate of unskilled labor.

The following simple model illustrates the conditions of optimal hiring of different educational categories of labor. The firm is assumed to be facing a labor market where the wage rate w is ruling for the unskilled category of labor. The skilled category is assumed to charge α above the wage rate of unskilled labor, depending upon the cost of education times the equilibrium rate of return to investment in education of the given duration. At the given cost and required return to education investments marginal productivity of the different categories of labor would have to be adjusted until the difference in marginal revenue product would be equal to the required rate of return compensating for the cost of education.

Although the growth of knowledge in the technological sense has often been regarded as exogenous to the firm, there is anyhow room for the formation of a knowledge strategy in the firm. The firm can decide on the optimal educational composition of the labor force.

We assume that the input of knowledge in the production process is represented by the amount of educated labor used (E). The production function Q (E, L) thus includes educated as well as uneducated labor (L). The production function is assumed to be concave and differentiable.

$$\underset{(E,L)}{\text{maximize}}\ V = pQ(E,L) - \omega\,(1+r)\,E - \omega L; \quad (6.1)$$

where p = product price;

ω = basic wage rate;

r = returns to education above the basic wage rate.

The optimality conditions are:

$$\frac{\partial V}{\partial E} = p\frac{\partial Q}{\partial E} - \omega(1+r) = 0; \qquad (6.2)$$

$$\frac{\partial V}{\partial L} = p\frac{\partial Q}{\partial L} - \omega = 0; \qquad (6.3)$$

$$\frac{\partial Q}{\partial E}\frac{1}{1+r} = \frac{\partial Q}{\partial L}; \qquad (6.4)$$

i.e. the use of educated labor should be adjusted, so that the marginal productivity of uneducated labor equals the marginal productivity of educated labor, discounted by the returns to education (above the standard wage rate).

As can be seen from this model the optimization criterion is the same as for any other factor input. The marginal value of the services provided by a certain educational category should be equalized with the marginal cost of that category of labor.

Research and Knowledge Investments in the Firm

The infrastructural conditions of industrial organization have changed since the days of Adam Smith and later classical analysts of industrial organization of the manufacturing firm.

In the stages of early industrialization infrastructure and other means of transportation and communication were notoriously slow and sparsely distributed in space. The labor force had little or no formal education and the technology of production was mostly pre-developed by tinkerers rather than being based on scientific foundations. Reliability and other qualities of the manufactured products were achieved primarily by massive inputs of raw materials, manual work and other energy. Production recipes and blueprints were generally at a low level of complexity, requiring limited formal education of foremen and workers. This does not imply that manufacturing work did not require working skills. In the process of "learning-by-doing" occupational skills most certainly developed, but this mostly happened without any substantial inputs of formal education.

The first economist to seriously discuss the use of basic or scientific knowledge in the firm was Joseph Schumpeter (1911). In Schumpeter´s theory knowledge was produced by independent scholars in universities and other higher institutions of education. Such scientific knowledge acquisition was assumed to be driven by curiosity rather than by expected profit motives. Around “the ivory towers” of learning, entrepreneurs were assumed to be scavenging for scientific findings to be profitably exploited by the entrepreneurs, financed by the capitalists operating in the financial markets. Thus, within this framework dependencies over time between science, technology patenting, innovation and diffusion of ideas can be illustrated by the following figure.

	Materialized	*Non-material*
Firm Specific	Innovation of idea in a prototype good **(Innovation)**	Patented idea **(Protection)**
Potentially Public	General distribution of the idea, embodied in the good **(Diffusion)**	Scientific idea **(Creation)**

Figure 1 The conceptual relations between creative scientific ideas, patenting, innovation and diffusion
(Dashed line indicates rare process.)

There is a necessary causal sequence from creativity to diffusion by sales of the new good in the market. There are also obvious uncertainties at the different stages of the process from scientific creativity to the completion of the new product to be sold in the market.

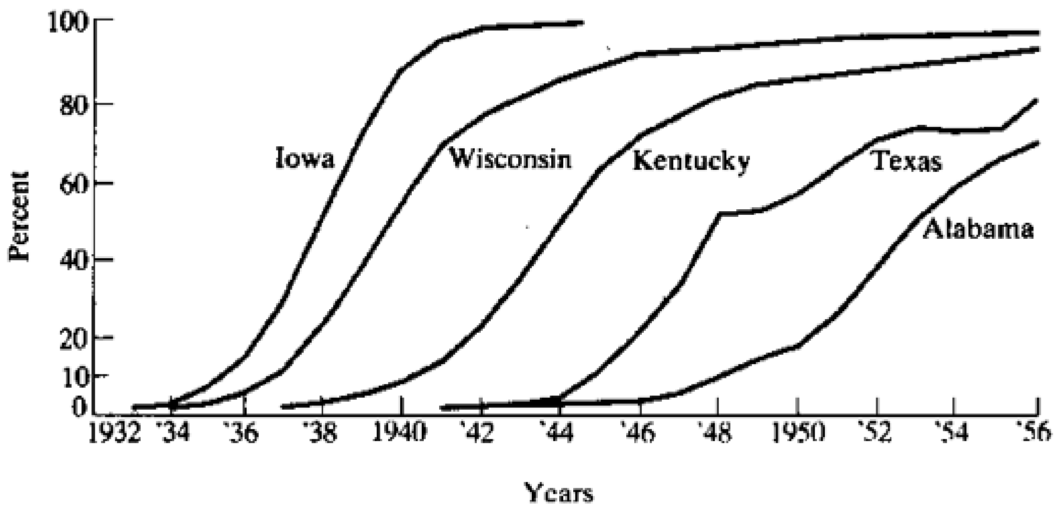
The speed of diffusion differs between ideas and their embodiment in tangible goods. Tangible goods of substantial complexity are evidently harder to imitate than e.g. simple food products such as hamburgers.

Innovation and Diffusion of Scientific Knowledge

The adoption of scientific knowledge by industry and the adaptation into new technologies by industrial research and development (R&D) are the two aspects of *innovation*. The innovation process has been extensively studied since the 1950s. The studies with the greatest impact were conducted by Zvi Griliches (1957) and Edwin Mansfield (1966).

Griliches followed the innovation-diffusion process of hybrid corn, where the new type of corn was the outcome of genetic research. Griliches could show that the innovation process followed an elongated s-curve as described in figure 3.

Diagram to be redrawn, based on the Hybrid corn article



Percentage of total acreage planted with hybrid seed

Zvi Griliches (1957), *Hybrid Corn: An Exploration in the Economics of Technological Change*
Econometrica, Vol. 25, No. 4 (Oct., 1957), pp. 501-522

Figure 6.2 . The innovation of hybrid corn in different states of the USA.

The innovation process of figure 3 follows essentially the same dynamic process, but with different times of introduction and in different speeds of innovation in different parts of the USA..

Mansfield conducted a study of innovations of agricultural machinery with similar results.

The simplest dynamic model consistent with these findings is the logistic differential equation model as described below.

$$\dot{Z}_i = \alpha Z_i (B - Z_i); \tag{5}$$

$\dot{Z}_i$ = growth of the number of firms adopting knowledge of type i, i.e. innovating knowledge of type i,

Z_i = number of firms that have already adopted knowledge of type i;

B = total number of firms capable of adopting knowledge of type i;.

An equilibrium requires $\dot{Z}_i = 0$, which can occur if $Z_i = B$ or $Z_i = 0$

$\dot{Z}_i = 0$ is an unstable equilibrium while $Z_i = B$ is stable.

The classical logistic equation (3) is a quadratic differential equation, showing that, once started, growth will increase towards a maximum and thereafter monotonously decline towards zero The total adoption of the new knowledge will thus approach the predetermined maximum level of use of a new technology. The shape of the logistic innovation curve is fully determined by two parameters, of which α denotes the speed of innovation/diffusion and B denotes the maximum number of firms that can adopt the new knowledge.

As it stands, economic interactions are not present in this dynamic model of innovations. A simple way of introducing economic interdependencies is by assuming that α is a function of relative prices. If the new knowledge is monopolistically controlled by the innovator, supported by a patent, demand will be a negatively sloped function of the price of the new knowledge. In the simple case of a linear demand function the logistic equation describing the innovation process would now become a *third degree* differential equation as formulated below in (6.5).

$$\alpha_i = \alpha_0 + \alpha_1 p_i \tag{6}$$

$$p_i = a_0 + a_1 Z_i; \tag{7}$$

$$\alpha_i = \alpha_0 + \alpha_1 (a_0 + a_1 Z_i) = A_0 + A_1 Z_i; \tag{8}$$

p_i = reactive price to knowledge of type i

Assumptions $A_0 > 0$ and $A_1 < 0$.

Thus $\dot{Z}_i = (A_0 + A_1 Z_i) Z_i (B - Z_i)$; (9)

$$\dot{Z}_i = A_0 Z_i (B - Z_i) + A_1 Z_i^2 B - A_1 Z_i^3; \quad (10)$$

$$\dot{Z}_i = A_0 B Z_i + (A_1 B - A_0 B) Z_i^2 - A_1 Z_i^3; \quad (11)$$

$\dot{Z}_i = 0$ implies that

$$A_1 Z_i^2 - (A_1 - A_0) B Z_i - A_0 B = 0 \quad (12)$$

This second-degree equation has one stable equilibrium solution Z_i^* with $0 < Z_i^* < B$.

With this formulation the dynamically stable equilibrium will be reached at a level below the full saturation point B. It is not in the interest of the monopolist, having secured a patent, to let the number of users of the new knowledge proceed beyond the level M in figure 3, denoting the temporary monopolistic saturation point. The whole cost of knowledge acquisition is sunk. Thus the point M corresponds to a level of prices at which a marginal revenue equals zero and the price elasticity of demand equals minus one. At some time (between ten and fifteen years) the pattern becomes invalid and the third degree part of the differential equation 11 would now be of the classical shape with the competitive equilibrium established with full saturation at the stable equilibrium point B.

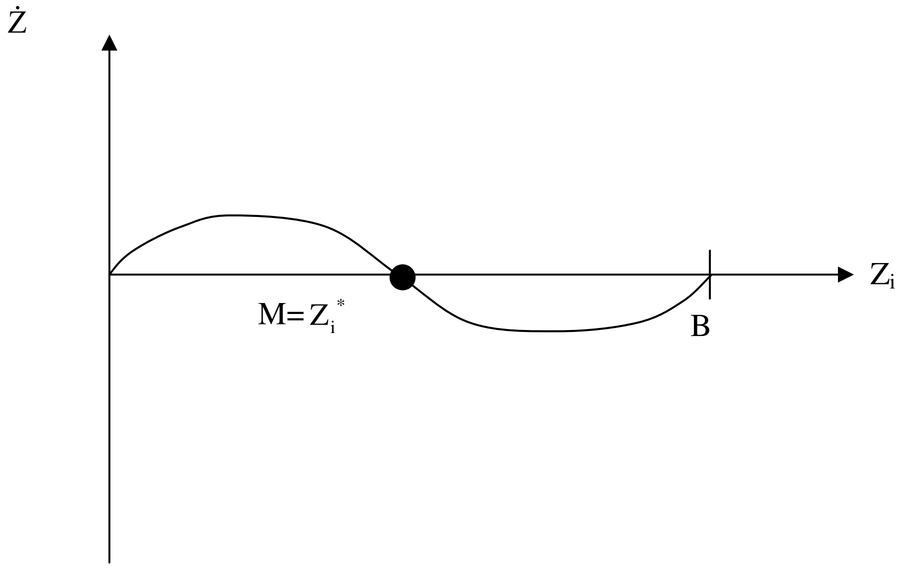


Figure 4 Rate of innovations of technology i with pricing feedback from the market

There are many examples of such a dynamic innovation process with an initial period of patent or copy rights, giving a temporary monopoly to the inventor (or the firm that has acquired the exclusive rights to the knowledge from the inventor). One recent example is the extensively used allergy drug Loratadin, which was initially monopolized under the name Claritin (or Clarityn). As soon as the patent rights expired, copies appeared on the market under the name Loratadin with different brand marks of genereca producing firms. The copies were sold at prices as low as one fifth of the monopoly price.

The Net Present Value of Research and Development

The investment in new knowledge in an industrial firm is to some extent similar to any other investment in e.g. machinery and other material capital. It is similar in the sense that the investments have to be based on evaluation of future incomes against early (or even immediate) costs. However, it is different from material capital investments because of the public nature of knowledge and the associated risks of competitive imitation by firms, who have not paid for the acquisition of the knowledge, necessary for the new production technology.

Let us for simplicity of exposition assume that a production firm (e.g. a pharmaceutical corporation) can acquire scientific research findings from medical and technological schools at the rate R. This knowledge will then be permanently available, generating increased production revenues at the level ΔQ. The net present value of a given scientific research input will thus be:

$$N = -C(R) + \int_{1}^{T} \Delta Q(R) e^{-rt} dt; \tag{13}$$

Assuming the impact of the purchased research to be valuable far into the future i.e. (T→∞), the net present value is simply, as stated in equation (14).

$$N = -C(R) + \frac{\Delta Q(R)}{r}; \tag{14}$$

where C(R) = cost of acquiring knowledge R.

$\Delta Q = \Delta Q(R)$ = increase of revenue as a function of R

N = net present value of R

The amount of scientific knowledge to be purchased by the firm should proceed until the net present value approaches 0 for the last amount of new knowledge purchased i.e. until:

$$\frac{dN}{dR} = -\frac{dC}{dR} + \frac{d(\Delta Q)}{dR} \cdot \frac{1}{r} = 0; \qquad (15)$$

$$\frac{d(\Delta Q)}{dR} = r \cdot \frac{dC}{dR};$$

Assume $\frac{dC}{dR} = p = 1$;

p = price per unit of knowledge

Thus $\frac{d(\Delta Q)}{dR} = r$; (16)

The marginal growth impact of increased new knowledge should equal the internal (required) rate of return.

Uncertainty and Risks of Knowledge Investments

Research and development projects are inherently uncertain. It is impossible to know beforehand what will be found at the end of a research process. The whole theory of statistical hypothesis testing is based on this fundamental uncertainty and consequently all hypothesis testing is formulated in terms of probabilistic arguments.

This fundamental uncertainty is also reflected in the organization of research and development at the industrial level. Research-based firms normally develop different strategies to efficiently handle their large uncertainties. One of the most important management problems in such firms is to generate a research portfolio that is large and diversified enough to reach an optimal combination of expected returns and risks. Many research projects are necessarily extended in time as well as in the requirement for indivisible resources. Sometimes collaboration with university departments is required, as is e.g. the case in the biotechnology and information technology sectors. Such an external relation ofte increases the uncertainties.

An example of the structure of uncertainty of bio-medical research is given in figure 6.5 where Pr_i = probability of success at stage i. These stages are irreversible and thus imply a problem of cumulative sunk costs. At each one of the 10 stages there is a possibility of stopping the project. There would be a cumulated probability of success of approx. 10 per cent, if we assume each probability of success (Pr) to be 0.8.

$Pr_i = 0.5$ for all i would imply a cumulated probability of success of less than 0.01 or $Pr^{10} = 0.5^{10} = 0.00098$.

The problem of very low probabilities of success of each research project can only be handled efficiently by increasing the scale of research-based firms in order to increase the number of uncertain research projects. The increasing scale of knowledge-based firms has been most evident in the pharmaceutical industry.

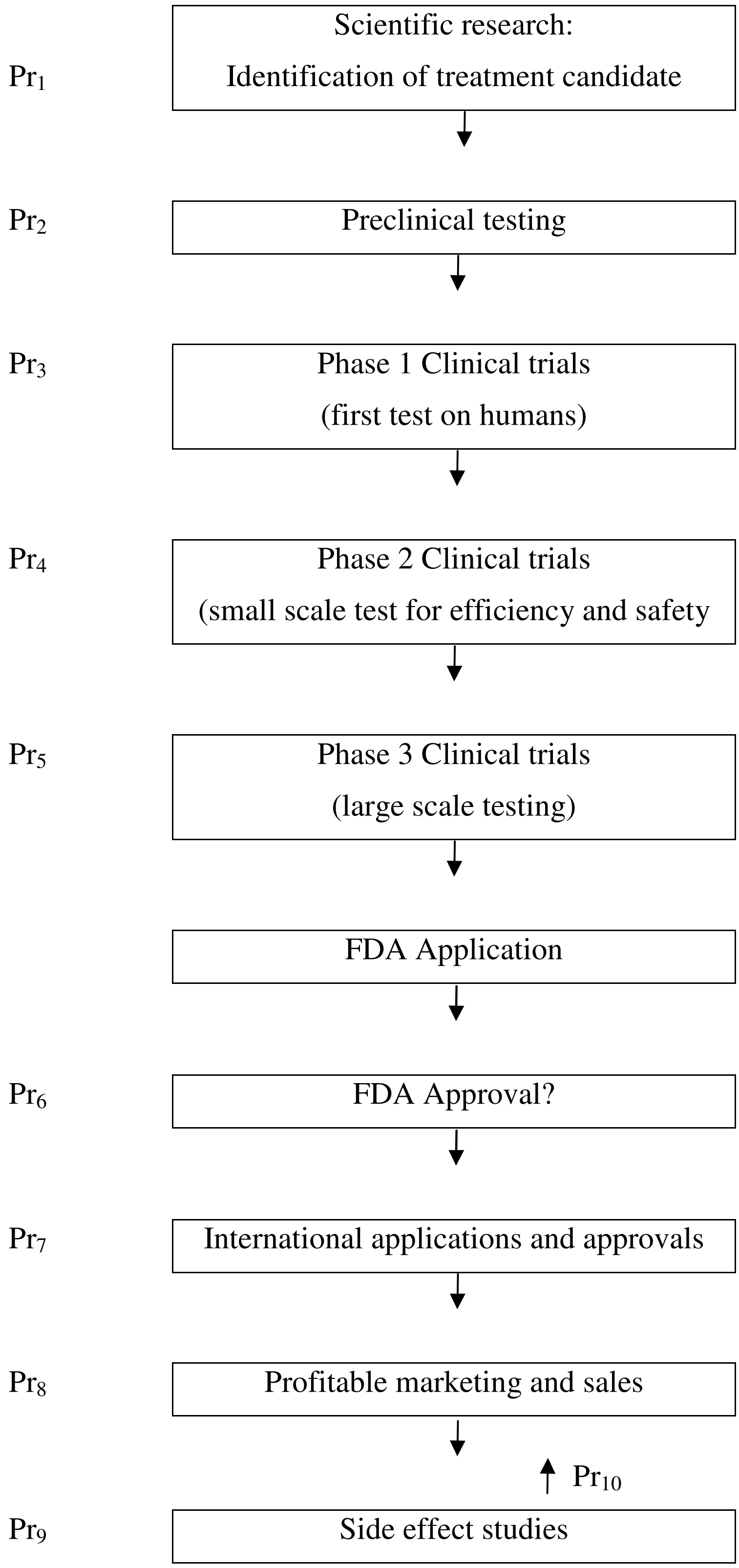


Figure 5 The probability structure of a pharmaceutical R&D-project

Net Present Value, Risk and Rates of Discount in R&D-projects

It is well known that there are great uncertainties associated with investments in knowledge by most industrial firms. As shown by Edwin Mansfield et.al. (1977) there is on the average an approximate fifty-fifty chance of technological success in research and development projects. However, the per cent change of success in the market place is much smaller and in the order of less than 20 per cent for a technologically successful project. The sluggish nature of returns from different new products in the knowledge-based firms can be accommodated in two alternative ways. The first procedure is to introduce the volatility of returns directly into the net present value calculation as shown in the following model.

Let us assume that we want to calculate the one period present value V which is

$$V = \frac{CEQ}{1 + r_\tau}; \qquad (17)$$

where CEQ = certainty equivalent revenue flow;

r_τ = risk-free rate of interest

Furthermore $V = \frac{Q}{r}$

According to the capital asset pricing model

$$r = r_\tau + \beta(r_m - r_\tau); \qquad (18)$$

where r = rate of return and discount of a given firm

r_m = rate of return of a fully diversified market portfolio

$$\beta = \frac{\text{covariance}(r, r_m)}{\text{variance}(r_m)} = \frac{\text{cov}(Q/V - 1, r_m)}{\text{var}(r_m)}; \qquad (19)$$

where β is a measure of the relative risk of investments in the firm. Q is uncertain, but not V.

Thus $\beta = \frac{\text{cov}(Q, r_m)}{V \cdot \text{var}(r_m)}; \qquad (20)$

$$\frac{Q}{V} r_\tau + \frac{\text{cov}(Q, r_m)}{V} \cdot \frac{(r_m - r_\tau)}{\text{Var}(r_m)}; \qquad (21)$$

$$V = \frac{Q - \text{cov}(Q, r_m) \cdot (r_m - r_\tau) / \text{var}(r_m)}{1 + r_\tau} \qquad (22)$$

Alternatively the capital asset pricing model (CAPM) can be used directly to determine the risk adjusted rate of discount r_i to be used in the discounting of future revenues into present value as follows.

Accordingly $r_i = r_\tau + \beta_i (r_m - r_\tau)$; (23)

where r_i = rate of discount (or return) of project i and $\beta_i = \frac{\text{cov}(r_i, r_m)}{\text{var}(r_m)}$

r_τ = the risk-free rate of interest,

r_m = the rate of return on the market portfolio

β_i is an elasticity, showing the percentage return following upon a 1 per cent increase of return in the market.

Estimates of β_i for knowledge or (R&D-) based firms are normally well above 1, indicating that investments in such firms are more volatile (or risky) than investments in a market portfolio.

The role of β_i in determining what risk-compensated rate of discount should be used in evaluating net present values of research and development projects is illustrated in the following diagram.

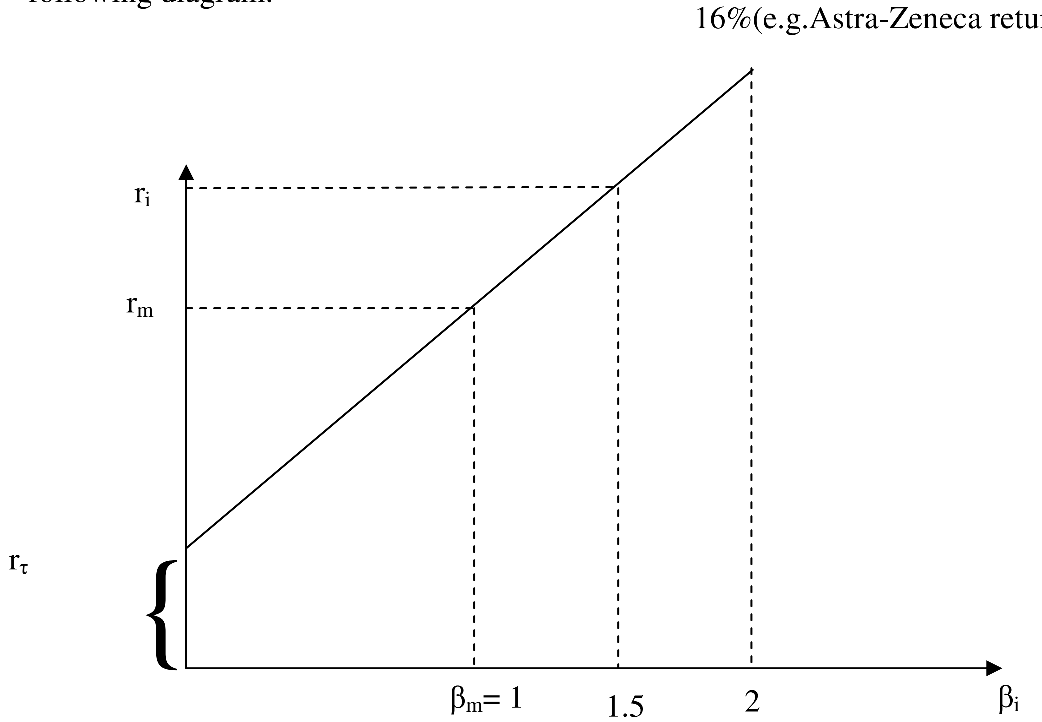


Figure 6 Risk and expected returns of different projects

For a knowledge-based firm like e.g. Astra-Zeneca, which in the first years of 2000 had an estimated β_i of 2, the risk-compensating rate of discount would be 16 per cent, i.e. with a risk-compensation of 12-13 per cent above the risk-free real rate of interest.

Comparative Estimates of Private Returns on Industrial Research Projects

One of the early, sophisticated studies of private returns to research investments is Mansfield et al (1977). In that study 36 industrial US research and development projects were included. The projects ranged from new products and processes to new control systems. The mean of the calculated returns was 39 per cent and the median value was 28 per cent, indicating a substantial skewness of returns.

The estimation of returns on research is often based on some simple econometric production function. The relation between private rate of returns and the production function is straight-forward.

Assume a Cobb-Douglas production function, including the impact of R&D-capital.

$$Y_{it} = A_{it} L_{it}^{\alpha} K_{it}^{\beta} \overline{R}_{it}^{\alpha} \varepsilon_{it}; \qquad (24) \text{ or}$$

$$\ln Y_{it} = \ln A_{it} + \alpha \ln L_{it} + \beta \ln K_{it} + \gamma \ln \overline{R}_{it} + \varepsilon_{it}$$

where Y_{it} = output of firm i at time period t

L_{it} = labor input in firm i at time period t

K_{it} = material capital input in firm i at time period t

$\overline{R}_{it}$ = knowledge capital input (equal to accumulated R&D-investments) in firm i at time period t

The estimated private return to knowledge capital $= \gamma \cdot Y_t / \overline{R}_{it}$.

Later estimates of private returns to R&D investments are seldom substantially different from Mansfield's estimates as indicated by the following table.

Table 7 Estimated rates of returns to R&D investments

	Rate of return (private)
Sveikauskas (1981)	0.17
Hall (1995)	0.33
Griliches and Lichtenberg (1984b)	0.34
Terleckyj (1980)	0.25
Scherer (1982)	0.29
Griliches and Lichtenberg (1984a)	0.30

Jeffrey I. Bernstein and M. Ishaq Nadiri (1997)have estimated private rates of returns on material capital and R&D investments for a number of knowledge-based manufacturing industries for the period 1964-1991 with results given by table .

Table 2. Private rates of return on physical and R&D capital 1964-1997. US manufacturing industries mean values (and standard deviations).

	Physical capital	R&D capital
Chemical products	0.208 (0.037	0.219 (0.028)
Non-electrical machinery	0.174 (0.050	0.212 (0.044)
Electrical products	0.167 (0.038)	0.202 (0.037)
Transportation equipment	0.182 (0.064)	0.178 (0.040)
Scientific instruments	0.199 (0.034)	0.254 (0.052)

Social Returns to Investments in R&D

Knowledge is, at least potentially, a public good. Private optimization of knowledge investments, based on private returns to R&D-investments, would thus be socially sub-optimal. The importance of knowledge capital accumulated elsewhere is obvious if the production function has the form $y_j = y_j\left(K_i, \bar{R}_1, \ldots, \bar{R}_i, \ldots, \bar{R}_n\right)$ with $\frac{\partial y_j}{\partial \bar{R}_i} > 0$.

Maximizing the sum of profits would then require that

$$\sum_j \frac{\partial y_j}{\partial \bar{R}_i} = r; \qquad (25)$$

where r = required rate of returns.

The discrepancy between the social and private returns is the main reason for the subsidies on investments in research. These subsidies are relatively largest in universities and other institutions engaged in scientific research. However, empirical studies of the discrepancies between private and social returns of industrial research indicate a need for increased subsidies also to such applied research.

In Mansfield et al (op.cit.) the average social returns to R&D is estimated to be approximately 100 per cent and the median value approximately 80 per cent. The social rate of return would thus be 2.5 to 2.9 times the private returns.

Late studies have shown the following ratios between social and private returns.

Table 3 Social relative to private returns on R&D-investments

Terleckyj (1980)	3.29
Scherer (1982)	2.55
Griliches and Lichtenberg (1984a)	1.37
Bernstein and Nadiri (1997)	
Chemical products	2.29
Non-electrical machinery	3.88
Electrical products	1.59
Transportation equipment	3.02
Scientific instruments	2.60

These studied indicate that social returns could be expected to be in the order of 2.5 times private returns. According to Jones' and Williams' (1998) analysis based on similar data the optimal allocation of research resources should be increased to four times the current level of R&D in the USA.

Knowledge Acquisition and Optimal Quality and Quantity

During the last three decades research and development has become an increasingly important strategic variable in decision making of firms. Rather than regarding new knowledge as a means of improving efficiency in producing an unchanged set of products, an increasing share of investment resources have been channeled to research into new products (and the associated methods of their production). In the jargon of R&D-economics, product-R&D has been given priority in strategic decision making. The term high-tech industry is strongly associated with a high share of investments, profits or value added being channeled into this type of knowledge investments. This redirection of investment strategies implies a reorientation from quantitative growth strategies to a focus on qualitative improvement of the products.

Measurement of quality is quite amenable to strict economic analysis of ideas developed by authors such as Morishima (1959), Hicks (1956), Lancaster (1966). However, the basic idea goes back to J. H. von Thünen (1826) in his analysis of the impact of location quality on the equilibrium price of land.

Each product (artifact or service) can be represented by an n-dimensional vector of characteristics. A bicycle is e.g. represented by some normal speed, energy requirement, comfort, etc. A car is represented by a different configuration of the same characteristics. In the terminology of Baumol concrete means of transportation are transformed into abstract modes. Technological changes would in this case be represented by Pareto-efficient increases of the characteristics of abstract modes or by the introduction of a completely new abstract mode with a new positively valued characteristic. It can further be assumed that each such new value of a characteristic is influenced by the accumulated level of knowledge, invested in research and development.

In von Thünen type models the willingness to pay for a product e.g. housing is determined by the location of the product. It is thus generally assumed that the accessibility to work, services etc determines the price of a unit of housing services. However, accessibility is only one of many important characteristics of “abstract modes of housing”. The quality of the neighborhood and characteristics of the housing unit itself can also be introduced as factors influencing the willingness to pay in terms of *bid-prices*. As the willingness to pay must necessarily increase with improved accessibility, an investment into improved infrastructure would then ordinarily imply an increase in the willingness to pay for housing of all the households, benefiting from the improved accessibility.

In the same way as the von Thünen type models of the pricing of housing is influenced by different characteristics of a unit of housing, other products can be assumed to be bid-priced by the vector of characteristics representing the product in a characteristics space.

In order to simplify matters we now assume the existence of a knowledge and information based firm producing new knowledge output with the help of inputs of knowledge and information. The inputs of knowledge are assumed to be proportional to the labor time of the knowledgeable labor. Examples of this type of firms are university departments, R&D institutes, software producers and consultancy firms.

We further assume that an increasing input of labour can either be used to increase the quantitative output of the knowledge firm or to improve the quality of the output, measured as increases in one or more of the valuable characteristics of the product. It is thus assumed that e.g. increasing the speed of execution of some computer software would require an increased input of knowledge that would necessarily have to be taken from other tasks of importance in the quantitative activity. There is thus a problem of allocation of knowledge labor between quantitative and qualitative tasks. To make the argument clear we now assume that a knowledge firm has a given amount of knowledge labor, L, to be allocated between qualitative and quantitative activities. It is further assumed (in line with out earlier discussion) that the work on improved product quality will influence the bid-price, p, of the users of the product, while the directly productive work will only influence the amount of products or the quantity produced. The firm is as usual assumed to be profit maximizing, which leads to the following optimization problem to be solved by management.

$$\max y = p(\rho \cdot \ell) \cdot q(\theta \cdot \ell) - \omega \cdot \rho \cdot \ell - \omega \cdot \theta \cdot \ell, \qquad (26)$$

where $\rho, \theta \geq= 0$ and $\rho + \theta = 1$;

ρ = the percentage share of total labor allocated to quality improvement tasks;

θ = the percentage share of total labor allocated to quantitative production;

ℓ = total knowledge labor available;

p = bid-price for a product

q = quantitative output.

For simplicity we assume the total amount of labor equal to unity. The p- and q-functions are assumed to be r-differentiable and concave. The conditions of a maximum are the following:

$$\frac{\partial y}{\partial \rho} = \frac{\partial p}{\partial \rho} \cdot q - \omega = 0\,; \qquad (27)$$

$$\frac{\partial y}{\partial \theta} = p \cdot \frac{\partial q}{\partial \theta} - \omega = p \frac{\partial q}{\partial (1-\rho)} - \omega = 0\,; \qquad (28)$$

$$\frac{\partial p}{\partial \rho} \cdot q = p \frac{\partial q}{\partial (1-\rho)}\,; \text{ or}$$

$$\frac{\partial p / \partial \rho}{p} = \frac{\partial q / \partial (1-\rho)}{q} \qquad (29)$$

This condition implies that the optimum percentage share of knowledge labor allocated to quality improvement should be

$$\rho = \frac{\varepsilon_{p,\rho}}{\varepsilon_{p,\rho} + \varepsilon_{q,\theta}}\,; \qquad (30)$$

The optimality conditions further indicate that the gradient of the bid-price function should be set equal to the marginal cost of labor divided by the **scale** of the operations of the knowledge firm. The allocation of labor to quantitative tasks is the conventional requirement that the marginal productivity of labor should equal the marginal cost.

Complexity, Knowledge and Value of Goods

Improving qualitative characteristics of many goods, e.g. computers, cameras or cars, requires not only the use of more capital and knowledge in the production of the goods. Increasing the computational speed of a computer, the pictorial precision of a camera or the energy efficiency of a car also requires new blue prints and associated production instructions. In an abstract sense blue prints and production instructions are programs of production similar to computer programs. And such programs are more or less complex. (Kolmogorov, 1966, Chaitin, 1967). The shortest possible program needed to generate all natural numbers is less complex than the shortest possible program generating all prime numbers. The complexity of a program for solving a given problem can be defined as *the minimal number of primitive instructions needed within the program in order to solve the given problem.*

It is possible to apply this definition to production of goods. A simple example can be given.

In some intuitive sense most people would agree that the object 1, ´pancake´ , is less complex than the object 2, ´bouillabaisse´. The definition of complexity given by Chaitin would then say that object 2 is more complex than object 1, because the minimal length of the recipe for cooking bouillabaisse would be much longer than the minimal length of a recipe for cooking pancake.

Obviously, a more complex object would entail larger accumulated effort, ceteris paribus, and would not be viable in the market, unless it could be sold at a higher price than a simpler object. Thus, increasing complexity of a good would imply increasing value (willingness to pay for) of the good.

Unfortunately, computer program (or mathematical theorem/proof) definitions of complexity is not enough for a definition of complexity of goods. A computer program is a symbolic recipe for production of a symbolic output from symbolic inputs. Any physical output is a set of physical and symbolic attributes produced by a set of physical inputs according to a symbolic recipe. Complexity of a good is thus at least three-dimensional:

1. Number of physical and symbolic attributes of the output compatible with a given value of a unit of the output to the user.
2. Number of different inputs needed to produce a unit of the output at the given value to the user.
3. Minimal length of the recipe for production according to requirements 1 and 2.

Thus, there must be at least a 3-dimensional representation o complexity in production theory.

Quality, Information and Research Contacts

The profitability of a firm is determined by the quality of the product as measured by the consumer willingness to pay and by the quantity produced of the product. The quantity of the product (at some given willingness to pay) is primarily determined by the use of different inputs, such as labor and other energy, information (for control of the production process), capital services and different types of materials, purchased locally or from other regions of the world. The optimal use of each one of these productive resources is determined by equalization of the value of the marginal productivity of each one of these resources with the relevant marginal cost of acquiring the resource. These are the *marginal productivity conditions* of the profit maximizing firm. They have formed the basis of micro-economic theory of production for more than a century. Still they provide the necessary conditions of optimal management of production units.

These conditions are, however, not satisfactory as means of analyzing the optimal behavior of the information and research dependent firm. In these increasingly important firms optimality must also include the determination of the level of product quality to be aspired. The quality of the product and thus the consumer willingness to pay for the product is determined by the quality of the input materials as well as a number of other obvious factors to be disregarded at this stage of the analysis. We will rather concentrate on the input of knowledge and information necessary to increase the internal complexity of the product. Much of the creative process of researchers and designers is primarily used up in substituting information and knowledge for energy and quantities of materials. The primary aim of this increase of information and knowledge is to achieve some consumer-relevant functional characteristics of the product. The final result is a higher willingness to pay. A relevant example of this substitution of materials and energy for complexity is the evolution of computing equipment and cars and other transportation equipment. Thus, we should expect the willingness to pay for a given commodity to be a function of the level of complexity of the product and indirectly to be a function of the inputs of information and knowledge into the product. In most fields of technology, research and development is distributed around the world and not concentrated to a single creative center.

Countries like Switzerland and Sweden, which have oriented much of its research resources into biomedicine, still has less than four per cent of the total research and development capacity of the world. Most of the knowledge of any industrial sector tends to be generated in other parts of the world, at least in the long run. This fact has important implications for the theorizing about future interregional, international and global interdependencies. The following, admittedly simplified, model of knowledge interdependency can indicate the structure of an improved theory of optimal management.

In this model we explicitly assume that the willingness to pay for a product of firm i (located in region i) is determined by the information I_{ji} and the knowledge R_{ji} received from region j; (j = 1, ..., n). We further assume that knowledge is of importance to product quality, only. The P and Q functions are both assumed to be concave and r-differentiable, everywhere.

$$\underset{\{I,K,L\}}{\text{Maximize}}\, G_i = P_i\left(I_{1i},\ldots,I_{ni};R_{ni}\right)\cdot Q\left(L_i,I_{ii}\right)$$

$$-\sum_j \tau_{ji}I_{ji} - \sum w_{ji}R_{ji} - w_iL_i - \tau_{ii}I_{ii}$$

$$\left.\begin{aligned} \frac{\partial G_i}{\partial L_i} &= P_i\frac{\partial Q}{\partial L_i} - w_i = 0 \\ \frac{\partial G_i}{\partial I_{ii}} &= P_i\frac{\partial Q}{\partial I_{ii}} - \tau_{ii} = 0 \end{aligned}\right\} \begin{matrix}\text{Marginal}\\ \text{Productivity}\\ \text{Conditions}\end{matrix}$$

$$\left.\begin{aligned} \frac{\partial G}{\partial I_{ji}} &= \frac{\partial P_i}{\partial I_{ji}}\cdot Q_i - \tau_{ji} = 0 \\ \frac{\partial G}{\partial R_{ji}} &= \frac{\partial P_i}{\partial R_{ji}}\cdot Q_i - w_{ji} = 0 \end{aligned}\right\} \begin{matrix}\text{Marginal}\\ \text{Interactivity}\\ \text{Conditions}\end{matrix}$$

In a theory of the networking firm there is not only the marginal productivity conditions for each one of the inputs influencing the optimal inputs of resources, regulating the scale of operations. There are also the quality oriented *marginal interactivity conditions* to be observed in the search for an optimal management strategy. The extent to which interaction ought to be by direct contacts or by transmission of information by communication links is an empirical issue, determined by the degree of substitutability between knowledge and information in the creation of product quality and consumer willingness to pay.

It should also be observed that the marginal interactivity conditions will tend to generate gravity-like interaction behavior in space.

The analysis shows that the following conclusions are warranted:

1. Interactivity is necessary for efficient scientific and industrial research
2. The interactivity conditions exhibit increasing returns to scale
3. Interaction between units of equal size are more favorable than between unequals
4. Increasing distance between research units decreases the advantages of interaction

Very little has been done in terms of empirical studies of R&D spillovers between firms in different locations. However, one example of international R&D spillover benefits is Bernstein and Nadiri (1997) with the following estimated benefit structure, see table 4.

Table 4 Balance of international R&D spillover benefit flows in 7 OECD countries, mean values of 1964-1991, (billions of US dollars, current priced)

to from	US	Japan	France	Germany	Italy	UK	Canada	Total
US		4.03	0.60	1.35	0.60	1.02	4.52	12.12
Japan	8.69		0.63	1.25	0.47	0.65	1.58	13.27
France	1.62	0.62		3.79	2.23	1.36	0.16	9.78
Germany	1.64	0.93	2.57		1.95	1.37	0.20	8.66
Italy	2.44	0.76	5.55	7.65		1.90	0.31	18.61
UK	2.06	0.86	1.49	2.52	0.90		0.48	8.31
Canada	10.20	0.89	0.22	0.39	0.19	0.51		12.40
Total	26.65	8.09	11.06	16.95	6.34	6.81	7.25	83.15

R&D spillover effects are obviously declining with distance and increasing with the sizes of the countries of the table.

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