CESIS Electronic Working Paper Series

Paper No. 177

Sources of Persistence in Regional Start-Up Rates
- evidence from Sweden

Martin Andersson* and Sierdjan Koster**
(‘CESIS and JIBS, ”Faculty of Spatial Sciences, University of Groningen)

April 2009
Sources of Persistence in Regional Start-Up Rates

- evidence from Sweden

by

Martin Andersson\textsuperscript{a} and Sierdjan Koster\textsuperscript{b}

martin.andersson@jibs.hj.se  sierdjan.koster@rug.nl

Abstract

The relationship between start-up rates and regional economic development has been studied rather extensively in recent years. Dynamics in start-up rates have however received considerably less attention. In this paper we analyze the persistence of start-up rates across Swedish regions over a decade and analyze the sources of persistence. We find overall persistence in start-up rates. Start-up rates of a decade earlier are able to explain over 40\% of the variation in current start-up rates across regions. The paper introduces and tests two mechanisms that can account for persistence in start-up rates across regions: (i) path-dependence in start-up activity, such that there is a response mechanism between previous and current start-up activity and (ii) spatially ‘sticky’ and durable determinants of start-ups. A dynamic panel analysis applying the system GMM estimator of lagged start-up rates on current start-up rates, confirms that persistence in start-up activity can be explained by both effects. Using transition probability analysis and quantile regression techniques, we also show that there is a regional dimension in persistence.

\textbf{JEL:} L26, R11, R12, O18

\textbf{Keywords:} entrepreneurship, start-ups, persistence, path-dependence, start-up dynamics, geography of entrepreneurship

\textsuperscript{a} Jönköping International Business School (JIBS), Jönköping, and the Centre of Excellence for Science and Innovation Studies (CESIS), Royal Institute of Technology, Stockholm.
\textsuperscript{b} Urban and Regional Studies Institute, Faculty of Spatial Science, University of Groningen.

\textbf{Acknowledgements:} We are grateful for comments and remarks from Börje Johansson, Pontus Braunerhjelm, Andreas Stephan, Aleid Brouwer and from seminar participants at the ERSA Congress in Liverpool 2008. Martin Andersson acknowledges financial support from the Swedish Governmental Agency for Innovation Systems (VINNOVA) and from the EU sixth framework program (MICRODYN project)
1. INTRODUCTION

The relationship between start-up rates and regional economic development has been studied rather extensively in recent years (see Van Praag and Versloot 2007 for a review of recent studies). Based on the ideas by Schumpeter, these studies contend that start-ups are a necessary condition for long-term regional economic development. Accordingly, there is a large interest in the geography of start-ups and its determinants among both academicians and policy makers. An area that has received considerable less attention, however, is the dynamics of start-up activity across regions over time. Questions pertaining to the extent to which the spatial distribution of start-up activity is persistent or change over time and the reasons for its change (or lack of change) remain largely unexplored in the literature. Such lines of inquiry is essentially about the dynamics of the geographical distribution of a dynamic phenomenon.

The current paper adds to the limited literature on the dynamics of start-up activity across regions over time. We provide further empirical evidence of persistence in start-up activity and try to untangle the forces behind the persistence by introducing and testing two not mutually exclusive mechanisms that can account for persistence in start-up rates. First, start-up rates are influenced by regional characteristics, such as income, educational level and population density. Since these factors are spatially ‘sticky’ and change in slow processes, start-up rates are expected to be persistent over time. Second, high levels of start-up rates in a region over a sequence of periods can generate demonstration effects which stimulate potential entrepreneurs to start new firms and create an ‘entrepreneurial climate’. Such a climate may reinforce future start-up activity. The first mechanism suggest that persistent differences in start-up rates across regions can be explained by persistent heterogeneity across regions. The second adds path-dependence in the start-up activity itself as a further explanation. This also suggests a regional dimension in persistence in the sense that persistence is expected to be stronger in regions with a more pronounced start-up activity.

Although persistence in start-up activity has been documented in previous studies, few studies focus directly on persistence as a phenomenon. The only study that deals directly with persistence in start-up rates is Fritsch and Mueller (2007), who find strong persistence in start-up rates across German regions. They focus on the policy implications and conclude that persistence in start-up rates indicates that entrepreneurship policies can only be successful if formulated for a long time horizon. In relation to previous studies, the novelty of this paper is that it assesses the sources of persistence and tries to conceptually embed persistence in start-up activity to the general discussion about patterns of change and path dependence in evolutionary economic geography in e.g. Martin and Sunley (2006) and Boschma and Frenken (2006). In this discussion, there are two aspects of path dependence that

---

1This result has been further empirically substantiated for the UK, Portugal and the Netherlands in a special issue in Small Business Economics (2008, issue 1) on the relationship between new firm formation and employment generation.
relate to start-up dynamics. On the one hand, start-ups can initiate new paths of economic development (Garud and Karnøe 2001). Start-ups thus introduce variety into a regional economy, which may induce long-term economic growth (Boschma and Frenken 2006). On the other hand, start-up behavior may itself be path dependent. Fritsch and Mueller (2007) and Van Stel and Suddle (2008) interpret persistence in start-up rates as a sign of path dependence. They do not, however, address the mechanisms that potentially drive persistence in start-up rates and how persistence conceptually relates to path dependence.

The empirical analysis consists of two parts. First, we empirically assess the two mechanisms for persistence in start-up activity, i.e. persistent regional characteristics that influence start-ups and path-dependence in start-up activity. For this purpose, we employ a dynamic panel model which include lagged start-up rates as additional regressors, and apply a system GMM estimator. In line with our argument, this model provides different reasons for correlation in start-up rates over time. One is directly through start-up rates in previous periods (in a dynamic panel data context often referred to as true state dependence). Another is through observable and unobservable heterogeneity across regions. Hence, the model allow us to test the two mechanisms that may drive persistence in start-up activity. Path-dependence manifests itself in such a way that start-up rates in previous periods influence current start-up rates. In the second part we assess the argument that there is a regional dimension in persistence, such that the strength in persistence is related to the level of start-up rates. We apply transition probability analysis and quantile regressions. The transition probability analysis examines whether the likelihood of switching state in terms of level of start-up rates in a period depends on previous states. The quantile regression technique is of semi-parametric nature and allows us to test whether the estimated marginal effect of lagged start-ups rates on current start-up rates differ across the distribution of start-up rates across regions.

The remainder of the paper is organized in the following fashion: Section 2 presents the theoretical framework. It starts by discussing the link between persistence and path dependence. It then goes on to discuss reasons for path dependence in start-up rates. Section 3 describes the data and illustrates persistence in start-up rates across Swedish municipality using data spanning a decade. Section 4 presents the results of the empirical analysis in which regional differences in persistence are explained. Section 5, finally, concludes and suggests future research avenues that emerge from the findings.
2. PATH DEPENDENCE AND PERSISTENCE IN START-UP RATES

2.1 Path dependence and persistence

Studies that find persistence in the regional distribution of start-up rates generally interpret this finding as a clear indication of path dependence in regional start-up dynamics (see e.g. Fritsch and Mueller 2007, Van Stel and Suddle 2008). The relationship may be more subtle though as becomes apparent when comparing the definitions of path dependence and persistence. Path dependence is the process in which later conditions are dependent on current ones (following Martin and Sunley 2006), such that development trajectories depend on initial conditions. Persistence, in contrast, is a statistical measure that addresses the lack of change in a phenomenon. Phenomena with high levels of persistence do not change much over time. Although path dependence implies that the level and direction of change is limited and piecemeal, it does not imply that change is necessarily small, particularly over longer periods of time. This suggests that interpreting persistence directly as a sign of path dependence is somewhat of an oversimplification. Rather, the relationship between persistence and path dependence should be interpreted indirectly: persistence in start-up rates is the result of path dependent processes that underlie regional start-up behavior. Viewing the relationship between persistence and path dependence in this way allows for the identification of different sources of persistence which suggests that there may also be regional differences in persistence: as path dependent processes that explain start-up rates can be regional distinct, it can be expected that there is also a regional dimension in persistence. To further elaborate the indirect relationship between persistence and path dependence it is then important to identify sources of persistence. This question is commonly asked in the literature concerning persistence in innovations at the firm level and this body of literature can inform the current discussion on persistence in start-up rates.

Raymond et al. (2006) give three alternative reasons for persistence in firms’ innovation activities. First, firms that innovate in a period incur sunk costs which provide a motive for innovation activity in subsequent periods as well. Secondly, innovation investments can be financed from the success of previous innovations. In this case, path dependence is apparent in the notion that success breeds further success. Thirdly, firms can learn from their own innovation activities, such that innovation activity is associated with dynamic scale economies or increasing returns. This makes innovation in one period dependent on innovation activity in the past.

Martin and Sunley (2006) list a similar division of reasons for path dependence. Firstly, path dependence can take the form of technological lock-in. A certain technological configuration becomes the accepted dominant design whereas other designs remain unused. The QWERTY-configuration on keyboards is arguably the best known example of this type (David 1985). Secondly, formal and informal institutions governing economic exchange develop over time in self-reinforcing processes.
and they are slowly changing (cf. North 1990). Martin and Sunley (2006) refer to this effect as ‘institutional hysteresis’. Thirdly, path dependence can be governed by dynamic increasing returns; path dependence is the result of positive feedback mechanisms including learning and the establishment of traded and untraded externalities (cf. Arthur 1994).

The latter two sources of path dependence as put forward by Martin and Sunley can also be interpreted in a regional context. Institutional hysteresis can be interpreted in a regional context in that it emphasizes that regional characteristics conducive for entrepreneurship are ‘sticky’ and durable and as such change in slow processes. In addition, dynamic increasing returns may be place-specific and create an ‘entrepreneurial climate’ that can account for persistence in start-up rates. The subsequent sections further elaborate these arguments.

2.2 ‘Sticky’ regional characteristics and persistence

In a system of differential equations where variables adjust at different time scales, it can be shown that the evolution and development trajectory of fast-adjusting variables is governed by the slower variables (Haken 1983). In a similar way, regional characteristics that are durable and change in slow processes play an important role in shaping the production possibilities and development trajectories of regions. Such characteristics can be given by nature or be created by different kinds of investments over time. A typical example of the latter is material infrastructure in the form of buildings, roads, airports and other investments in durable interaction capacity. Johansson and Wigren (1996) introduce the concept of the ‘production milieu’ as a comprehensive term for this kind of durable and spatially ‘sticky’ regional attributes.

However, several production factors that are essentially mobile can be maintained to be part of a region’s production milieu. This applies to many of the regional characteristics the literature shows influences start-up activity. There is plenty of evidence that regional characteristics reflecting local demand- and supply-side conditions, such as the education level, innovation activity, market-size, industry structure, agglomeration economies, do indeed influence the rate of new firm formation in regions (Verheul et al. 2001). These characteristics typically change in slow processes and can be claimed to be part of a region’s production milieu. Educated and skilled workers are for example mobile production factors, but ample research show that educated and skilled workers typically concentrate in already human capital-intensive locations (see inter alia Glaeser et al. 2003, Florida 2002, Berry and Glaeser 2005). A region’s level of human capital evolve in path dependent processes where high levels in the past entail high levels in subsequent periods. From this perspective the experiences, knowledge and competence of the labor force in a region can from an aggregate perspective be considered as a durable attribute. Moreover, the production milieu of a region also

---

2 This is known as Haken’s ‘slaving principle’.
comprises agglomeration economies. Such place-specific external scale economies typically evolve in self-reinforcing and self-organized processes over time and, once materialized, constitute a durable attribute (cf. Krugman 1996).³

Start-up rates across regions are in this sense governed by durable and spatially sticky variables. In view of this one would thus a priori expect persistence in start-up rates, simply because of the durability of the determinants. This does not mean that levels of start-up rates do not change from year to year. Exogenous shocks may change the opportunities for start-ups leading to year-to-year differences in start-up rates. However, despite possible temporal fluctuations the regional distribution in the start-up rates is expected to be persistent over time.

2.3 Path dependence in start-ups and persistence

The second source of persistence in start-up rates is path-dependence in the start-up process itself, such that start-up activity in current periods is partly a response to the same phenomenon in previous periods. There are two main reasons for a positive response mechanism from a region’s recent history of start-up activity to current start-ups.

Firstly, a high level of start-up activities generates new entrepreneurial opportunities (Holcombe 2003). When new ideas are materialized by entrepreneurs in a region in the form of new products or services produced by new firms, these can generate new entrepreneurial opportunities to explore (Audretsch and Keilbach 2004b). Frenken and Boschma (2007) formulate a dynamic theoretical model of rank-size distributions along these lines. The model builds on a branching process with product divisions as units of analysis. A new product division is started when either a firm or an employee decides to commercialize an innovation. In accordance with Schumpeter (1934) an innovation is defined as a recombination of existing products. This means that with each new product division, the number of possible innovations increases non-linearly. The growth of innovations (and the related product divisions) can be interpreted as path dependence in the process itself. Endogenous growth theory as formulated by Romer (1990) is built on similar premises.⁴ In conclusion, this suggests that a high number of start-up creates economic diversity and opportunities, which can materialize in new start-ups.

Secondly, start-up activity may stimulate a local entrepreneurial ‘climate’.⁵ Regions that for whatever reasons have high levels of start-up rates over a sequence of periods may develop an entrepreneurial climate which fuel the start-up of new firms in consequent periods (cf. Wagner and

³ The observed persistence in city-size distributions and spatial hierarchies is a significant illustration of this durability.
⁴ In these models the accumulated knowledge is the basis for new innovation and ideas. New knowledge adds to the stock of existing knowledge and increase the innovation potential of the economy.
⁵ Audretsch and Keilbach (2004a) label regions with a favorable entrepreneurial climate as being rich in entrepreneurial capital.
Entrepreneurs starting new firms can serve as role models for potential entrepreneurs. A high frequency of role models in a region may generate ‘demonstration effects’ such that potential entrepreneurs are stimulated to materialize an idea in the form of a new firm (Henrekson and Stenkula 2007). Johannisson (1983, 1984) provides a discussion and illustration of this kind of effect in the Gnosjö region in Sweden. Guiso and Schivardi (2005) argue that when more entrepreneurs are active in a region, people have higher chances of acquiring entrepreneurial skills. Entrepreneurial ‘talents’ may thus differ across locations due to differences in learning opportunities. In their model, such learning opportunities are related to previous start-up activity since they are assumed to depend on the intensity of entrepreneurs in a given region. These ideas are on par with the notion of ‘imitative entrepreneurs’ discussed by Baumol (1993). Moreover, there are general arguments in the literature that local culture (or informal institutions) may influence start-up behavior. Westlund and Bolton (2003) discuss the link between social capital and entrepreneurship. Etzioni (1987) points to the role of societal legitimation for entrepreneurship. Davidsson and Wiklund (1997) show that local ‘values’ and ‘beliefs’ have an impact on start-up activity. Generally, one could expect that a region’s history of start-up activity influence these types of factors.

Figure 1 summarizes the argument of the sources of persistence in start-up rates across regions outlined in the previous and the current section. The regional attributes that explain start-up rates change in slow processes and consequently, start-up rates should change slowly over time. The second factor is self-reinforcing mechanisms, which imply that start-up activity in the current period is partly a response to the same phenomenon in previous periods. It is evident that any change process driven by slowly changing regional characteristics and invariant feedback (or self-reinforcing) mechanisms will be persistent.

Figure 1. Conceptual model

Accepting the existence of a feedback mechanism between current and past start-up activity in a region involves accepting that on top of durable and spatially sticky characteristics there is an additional enduring advantage for regions that have shown high start-up rates in previous periods. This

---

6This argument is similar to learning-by-doing in innovation processes as an explanation for persistence innovation at the firm-level (Raymond et al. 2006).
leads to the expectation that the strength of persistence in start-up rates is dependent on the level of start-up rates. Regions with high start-up rates over a sequence of periods are assumed to have a strong entrepreneurial culture, or well-developed entrepreneurial capital, which induces persistence in start-up rates. This does not necessarily apply to the actual level of the start-up rates, but it does apply to the regional distribution of start-up rates.

The paper continues along the lines sketched in the above figure. First, we investigate persistence in start-up rates in Swedish regions (Section 3). The second step of the analysis searches for differences in persistence according to the level of regional start-up (Section 4).

3. ILLUSTRATING PERSISTENCE IN START-UP RATES

3.1 Data and setting the scene

For this study we use data that originate from Statistics Sweden and provide information on the number of start-ups in each municipality in Sweden from 1994 until 2004. In these data, a start-up is defined as a new establishment. The data material separates between truly new establishments and new establishments that resulted from reorganizations or change of ownership structure. In the current analysis only the first type of start-ups has been used, i.e. truly new establishments. A new establishment is either an entirely new firm or a new branch started by an expanding firm. Unfortunately, these types cannot be separated in the data.

![Figure 2. Start-ups and GDP per capita in Sweden 1994-2004 (ratio between the level in each period and the average level during 1994-2004).](image)

Figures 2 and 3 give a descriptive overview of aggregate start-up dynamics in Sweden in the study period. These figures present the context in which the analysis must be placed. Figure 2 presents the co-variation between the total number of start-ups, the start-up rate (total number of start-ups divided
by labor-market population) and GDP per capita over time. Both the start-up rate and the GDP per capita rate in each year are expressed as the ratio between the period’s value and the average value over the whole period, such that they are measured on the same scale. The first conclusion is that start-up activity has fallen during the study period. At the same time the GDP per capita has increased steadily. One reason for the decrease in start-up activity since 1994 can be that 1994 marked the end of a recession in the early 1990s. This period was associated with high unemployment rates which could have pushed necessity entrepreneurship. Improved economic conditions and a consequent recovery of the labor-market in 1994 and onwards may have impeded start-up activity. On average over the period 1994-2004 there are about 32 000 start-ups per year.

Figure 3 decomposes the start-ups per year in three industry aggregates: (i) manufacturing, (ii) low-end services, (iii) high-end services.\(^7\) It is evident from the figure that most start-ups are in service industries and a declining share of the start-ups is in manufacturing industries. During the period 1994-2004 the number of start-ups in high-end services has increased its share most. This is in line with developments in other European countries.

![Figure 3. Distribution of start-ups across three industry aggregates per year 1994-2004.](image)

3.2 Persistence of start-up rates in Swedish municipalities

This section addresses the persistence in start-up rates across Swedish municipalities. The illustration is akin to the one in Fritsch and Mueller (2007) allowing for a comparison with their results based on German data. Following the labor market approach in the measurement of start-up rates (Audretsch and Fritsch 1994), the start-up rate in each municipality is calculated by dividing the total number of

\(^7\) Low-end services are defined by NACE code 50-64 and includes retail, wholesale, hotels, restaurant and repair shops. High-end services are defined by NACE code 65-99 and include advanced producer services, R&D institutions, etc.
start-ups in a municipality with its labor market population. As a first inquiry to persistence in start-up rates we ask whether the observed change in Swedish overall start-up rates during the period 1994-2004 was associated with a change in the distribution of start-up rates across Swedish municipalities. Figure 4 presents the estimated kernel density for start-up rates across Swedish municipalities in 1994 and 2004. It is evident from the figure that the distribution of start-up rates among municipalities has a similar shape in 1994 and 2004. Corresponding to the overall decline in Swedish start-up rates the curve describing the distribution has moved to the left in 2004 compared to 1994, but the shape of the curve remains intact. This indicates that the overall decline in start-up rates at the aggregate level corresponds to a fairly even decline among different locations, such that the distribution of start-up rates among location remains.

![Kernel density estimates of the distribution of start-up across municipalities in Sweden 1994 and 2004.](image)

**Figure 4.** *Kernel density estimates of the distribution of start-up across municipalities in Sweden 1994 (Entr_rate_1994) and 2004 (Entr_rate_2004).*

A distribution can however remain stable over time intervals even though individual locations shift positions during the time interval. Figures 5a and 5b plot regional start-up rates in 2004 against start-up rates in 2003 and 1994 respectively. Each figure includes a fitted line estimated with ordinary least squares (OLS). The figures illustrate that there is strong persistence in start-up rates among Swedish municipalities. This indicates that not only the distribution of regional start-up rates remains stable over time (as indicated in Figure 4), there is also an evidently positive relationship between start-up rate in period \( t \) and period \( t-1 \) (Figure 5a). Figure 5b shows that this relationship is not a short-term phenomena; the positive relationship between start-up rate in period \( t \) and \( t-10 \) is only slightly weaker.
Figures 5a and 5b. Relationship between start-up rates $t$ and $t-1$ (left) and $t-10$ (right) across Swedish municipalities.

Table 1 offers a more complete overview of persistence of start-up rates over time. It shows the coefficients of simple OLS regressions with the start-up rate in 2004 as dependent variable and the lagged start-up rates as independents. The results support the idea that there is large degree of persistence in the regional distribution of start-up rates. Over a decade the coefficients only drop slowly and the start-up rate with the greatest lag ($t-10$) still explains over about 50% of the variance of the start-up rate in the base year (2004). These results concur with the results of Fritsch and Mueller (2007). They find a similar pattern, although the explanatory power of lagged start-up rates seems to be somewhat stronger across regions in Germany.

Table 1. OLS regressions of start-up rate in $t$ (2004) across Swedish municipalities with start-up rates with different lag lengths as independent variables.

<table>
<thead>
<tr>
<th>Start-up rate (t)</th>
<th>Start-up rate (t = 2004)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start-up rate (t-1)</td>
<td>0.809***</td>
</tr>
<tr>
<td>Start-up rate (t-2)</td>
<td>0.844***</td>
</tr>
<tr>
<td>Start-up rate (t-3)</td>
<td>0.931***</td>
</tr>
<tr>
<td>Start-up rate (t-4)</td>
<td>0.833***</td>
</tr>
<tr>
<td>Start-up rate (t-5)</td>
<td>0.829***</td>
</tr>
<tr>
<td>Start-up rate (t-6)</td>
<td>0.790***</td>
</tr>
<tr>
<td>Start-up rate (t-7)</td>
<td>0.853***</td>
</tr>
<tr>
<td>Start-up rate (t-8)</td>
<td>0.763***</td>
</tr>
<tr>
<td>Start-up rate (t-9)</td>
<td>0.668***</td>
</tr>
<tr>
<td>Start-up rate (t-10)</td>
<td>0.725***</td>
</tr>
</tbody>
</table>

R-square (average across the 10 estimations) 0.52

Note: The table reports estimated parameters of start-up rate with different lag lengths. Separate regressions for each lag length. *** $p<0.01$
Table 2 looks for a possible temporal dimension in persistence by showing the crude correlations between the start-up rate in one year and in the previous years. Again persistence is clearly visible in the data. In addition, the table indicates that persistence seems rather time invariant. Regardless of the base-year, there are high and only gradually declining correlations between start-up rates and their lagged counterparts.

The main conclusion of the results presented in the above is that the regional distribution of start-up rates is persistent over time. This can indeed be interpreted as an argument that policy measures geared towards promoting the start-up of new companies should have a long time-horizon. It also gives an interesting empirical example of how relatively static spatial distributions of economic phenomena can be explained with dynamic indicators (cf. Frenken and Boschma 2007). If a spatial distribution is to be stable over time, the distribution of related dynamic indicators (such as start-up rates) needs to mimic the existing distribution. The persistence of start-up rates is an example of such a phenomenon.8

Table 2. Correlations between start-up rates 1994-2004 across Swedish municipalities.

<table>
<thead>
<tr>
<th>Start-up rates</th>
<th>t-1</th>
<th>t-2</th>
<th>t-3</th>
<th>t-4</th>
<th>t-5</th>
<th>t-6</th>
<th>t-7</th>
<th>t-8</th>
<th>t-9</th>
<th>t-10</th>
</tr>
</thead>
<tbody>
<tr>
<td>t=2004</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t=2003</td>
<td>0.74</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t=2002</td>
<td>0.72</td>
<td>0.75</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t=2001</td>
<td>0.73</td>
<td>0.72</td>
<td>0.74</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t=2000</td>
<td>0.72</td>
<td>0.74</td>
<td>0.72</td>
<td>0.74</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t=1999</td>
<td>0.70</td>
<td>0.72</td>
<td>0.73</td>
<td>0.72</td>
<td>0.73</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t=1998</td>
<td>0.67</td>
<td>0.71</td>
<td>0.72</td>
<td>0.73</td>
<td>0.71</td>
<td>0.73</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t=1997</td>
<td>0.68</td>
<td>0.70</td>
<td>0.72</td>
<td>0.74</td>
<td>0.74</td>
<td>0.72</td>
<td>0.73</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t=1996</td>
<td>0.68</td>
<td>0.67</td>
<td>0.69</td>
<td>0.72</td>
<td>0.72</td>
<td>0.73</td>
<td>0.70</td>
<td>0.73</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>t=1995</td>
<td>0.68</td>
<td>0.72</td>
<td>0.67</td>
<td>0.72</td>
<td>0.73</td>
<td>0.74</td>
<td>0.76</td>
<td>0.74</td>
<td>0.73</td>
<td>1.00</td>
</tr>
<tr>
<td>t=1994</td>
<td>0.70</td>
<td>0.76</td>
<td>0.73</td>
<td>0.67</td>
<td>0.75</td>
<td>0.76</td>
<td>0.74</td>
<td>0.80</td>
<td>0.76</td>
<td>0.75</td>
</tr>
<tr>
<td>Mean</td>
<td>0.73</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
<td>0.77</td>
<td>0.78</td>
<td>0.79</td>
<td>0.81</td>
<td>0.83</td>
<td>0.88</td>
</tr>
</tbody>
</table>

4. Sources of persistence and variation across regions

The previous section focused on illustrating the existence of persistence in start-up rates across Swedish regions. This section assesses the follow-up questions of (i) the sources of persistence and (ii) whether there are any regional differences in the level of persistence itself. Section 4.1 addresses (i) whereas Section 4.2 addresses (ii).

8 Fritsch and Mueller (2007) find that both the levels of start-up rates and changes in these levels are explained by the same variables, which adds to this idea.
4.1 Sources of persistence in regional start-up activity – an empirical test

The conceptual framework introduced in Section 2 suggests a two-pronged explanation for persistence in start-up rates. First, slowly changing regional characteristics pertinent to the explanation of start-ups suggest persistence in start-up rates across regions. Second, persistence in start-up activity can also be due to path-dependence in the start-up process itself, such that start-up activity in current periods is partly a response to the same phenomenon in previous periods. These two explanations are not mutually exclusive; they are rather complementary. In order to test both effects we estimate a dynamic panel data model:

\[ S_{it} = \gamma_t S_{i,t-1} + \ldots + \gamma_{t-n} S_{i,t-n} + x_i^r \beta + \alpha_i + \mu_t + \epsilon_{it} \]

where \( S_{it} \) is the start-up rate in municipality \( i \) in year \( t \), and \( t-n \) denotes lag length. \( x_{it} \) is a matrix of regional characteristics assumed to influence start-up activity. \( \alpha_i \) denotes time-invariant municipality-specific effects, \( \mu_t \) time-effects and \( \epsilon_{it} \) is an error term. In accordance with our theoretical framework, the dynamic panel model in (1) provides different reasons for correlation in start-up rates over time (Cameron and Trivedi 2009):

(i) through start-up rates in previous periods. Current start-ups is partly a response to the municipality’s start-up rates in the recent history, suggesting path-dependence in start-up activity. Such a response mechanism is motivated by the arguments in Section 2.3.

(ii) through observed heterogeneity in the form of factors in \( x_{it} \) assumed to influence start-up rates in a region.

(iii) through unobserved time-invariant heterogeneity captured by municipality-specific effects, \( \alpha_i \).

The dynamic panel model in (1) thus allow us to test the empirical relevance of the arguments outlined in Section 2 and summarized by Figure 1. The model incorporates both regional characteristics that influence start-up rates and a response mechanism from previous to current start-up rates. In addition we control for unobserved heterogeneity.

Our choice of variables in \( x_{it} \) is based on previous literature on the determinants of start-up rates in regions. Verheul et al. (2001) provide a division of variable types that are pertinent determinants for start-ups rates across regions. They distinguish between supply, demand and
institutional effects. In the present analysis, the supply-side is represented by the educational level of the population, measured as the share of the population with higher education (≥3 years of university education). In addition, the regional share of service industry firms enters as an indicator of the economic structure. Both variables are expected to positively influence start-up rates. The demand side enters the regression with special attention for possible spatial effects in demand. We apply an accessibility measures to markets outside each municipality based on exponential distance decay, which satisfy criteria of consistency and meaningfulness (Weibull 1976). Specifically, demand-side conditions in municipality \( i \) in time \( t \) is approximated by \( \sum \text{GRP}_j \exp\left\{-\lambda t_{ij}\right\} \), where \( \text{GRP} \) denotes the gross regional product in municipality \( j \), \( \lambda \) is a distance friction parameter and \( t_{ij} \) is the time distance between \( i \) and \( j \) in terms of traveling time by car. In addition to the composite measures of demand, income is also included as it is has been shown to be an important variable for explaining variation in start-up rates (Reynold et al. 1995). The interpretation, however, is ambiguous. Income can be seen as a demand variable indicating market potential of regions. It can, however, also be interpreted as a supply-side variable indicating availability of start-up capital. Finally, it can negatively influence start-up rates as high income levels induce high opportunity costs for becoming an entrepreneur. The median income level is used as the distribution of income in regions is skewed.

All these variables are stable over time, which reflects the argument of slowly changing determinants of start-up activity. We also include a dummy variable for metropolitan areas which comprise Sweden’s three major cities (Stockholm, Göteborg and Malmö).

To estimate the model in (1), we employ the two-step system GMM estimator. The system GMM estimator has a set of attractive properties and has been shown to often improve over alternative estimators (Blundell and Bond 1998, Blundell et.al 2000), specifically the Arellano-Bond estimator (Arellano and Bond 1991). The Arellano–Bond estimator is based on first-differencing the data and instrumenting all potentially endogenous variables with their own levels. In our empirical context with regional characteristics that change slowly over time, the system GMM estimator is particularly attractive. Blundell and Bond (1998) show that the first-difference GMM estimator behave poorly when the variance of the individual-specific fixed effects, \( \alpha_i \), is large compared to the variance of \( \varepsilon_i \). When series are persistent such that variables change slowly over time, lagged levels are weak instruments for the first differenced variables. As proposed by Blundell and Bond (1998), the system GMM adds moment conditions and combines first-differences and levels in that it is based on a system of first-differences instrumented on lagged levels and of levels instrumented on first differences. Persistent series in an important circumstance in which the system GMM estimator has superior performance over the first-difference GMM estimator (Blundell and Bond 1998).

Table 3 presents the results from the estimation. The table reports results obtained with pooled OLS, the within (fixed effects) panel estimator and the two-step system GMM estimator. The
estimated parameters of the lagged start-up rates obtained with the system GMM estimator pass the ‘bounds test’ (Roodman 2006) and lie between the pooled OLS (upward bias) and the within estimates (downward bias). In the system GMM estimation, standard errors are estimated with the Windmeijer (2005) correction and are robust to heteroscedasticity. All variables except the time dummies, the dummy for metropolitan areas and external market size are specified as endogenous. The estimations include three lags of the dependent variable and the test statistics regarding autocorrelation and validity of the instruments are satisfactory. First, the tests for first- and second-order autocorrelation, AR(1) and AR(2), indicate no problem. The null hypothesis in the AR(1) test is rejected and the null hypothesis in AR(2) test is not rejected. Second, regarding validity of the instruments the Hansen test for overidentifying restrictions is satisfactory. The null hypothesis is not rejected. The Hansen test is robust but can be weakened by many instruments. In our model there are 260 instruments which is lower than the number of cross-section units (286), and the number of instruments should not be an issue (cf. Roodman 2006). The estimated parameters obtained with the two-step system GMM estimator are conditional on municipality-specific fixed effects and account for endogeneity associated with the regressors. It allows for a more causative interpretation than pooled OLS and within models. We focus on the results from the two-step system GMM estimator.9

The results in the table are supportive for that start-up rates in a municipality over time can be explained by both regional characteristics and previous start-up rates. The lagged start-up rates are statistically significant and the results suggest a causal effect from previous to current start-up activity in a municipality, which is consistent with path-dependence in start-up activity. Moreover, the regional characteristics are significant and confirm the findings in previous literature. The education intensity, market-size and share of services in the local industry is positive and statistically significant. Income and employment share is associated with a negative parameter estimate, but are not statistically significant. In Appendix B we present the estimated parameters of these variables when lagged start-up rates are excluded, and it does not change the pattern in Table 3.

---

9An issue with empirical analyses of spatial data is problems associated with spatial dependence (Anselin 1988, Anselin and Florax 1995). In our case, start-up rates across municipalities are spatially dependent. Appendix A presents a Moran’s scatterplot for start-up rates in 2004. The estimations in Table 3 does not address issues associated with spatial dependence in a direct manner. The market-size variable could be argued to partly capture spatial effects across municipalities (cf. Andersson and Gråsjö 2009), but we know of no formal test for assessing this issue in a dynamic panel data context. As a rough check, we have estimated a cross-section model which include lagged start-up rates as regressor. In this case the Moran’s I test suggest no problem with spatial autocorrelation. Moreover, spatial lag and error models provide identical results as an ordinary OLS model.
Table 3. Dynamic panel estimations. Estimated parameters of explanatory variables in a regression equation with start-up rates in Swedish municipalities as dependent variable.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Pooled OLS</th>
<th>Within (fixed effects)</th>
<th>Two-step System GMM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lagged start-up rate (t-1)</td>
<td>0.307***</td>
<td>-0.059**</td>
<td>0.264***</td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.028)</td>
<td>(0.028)</td>
</tr>
<tr>
<td>Lagged start-up rate (t-2)</td>
<td>0.222***</td>
<td>-0.069**</td>
<td>0.187***</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.028)</td>
<td>(0.035)</td>
</tr>
<tr>
<td>Lagged start-up rate (t-3)</td>
<td>0.269***</td>
<td>-0.030</td>
<td>0.227***</td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.028)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>Education intensity</td>
<td>0.017***</td>
<td>0.013</td>
<td>0.018**</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.017)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Services (share)</td>
<td>0.431***</td>
<td>0.675</td>
<td>0.814***</td>
</tr>
<tr>
<td></td>
<td>(0.131)</td>
<td>(0.758)</td>
<td>(0.321)</td>
</tr>
<tr>
<td>Market-size</td>
<td>0.678***</td>
<td>4.053</td>
<td>1.461***</td>
</tr>
<tr>
<td></td>
<td>(0.259)</td>
<td>(2.472)</td>
<td>(0.498)</td>
</tr>
<tr>
<td>Income (log)</td>
<td>0.192</td>
<td>-1.736***</td>
<td>-0.909</td>
</tr>
<tr>
<td></td>
<td>(0.411)</td>
<td>(0.467)</td>
<td>(0.853)</td>
</tr>
<tr>
<td>Employment share</td>
<td>-1.051**</td>
<td>-2.285</td>
<td>-0.435</td>
</tr>
<tr>
<td></td>
<td>(0.481)</td>
<td>(1.555)</td>
<td>(0.906)</td>
</tr>
<tr>
<td>Metropolitan dummy</td>
<td>0.185*</td>
<td>-</td>
<td>0.450</td>
</tr>
<tr>
<td></td>
<td>(0.105)</td>
<td>-</td>
<td>(0.309)</td>
</tr>
<tr>
<td>First-order autocorrelation AR(1), (p-value)</td>
<td>-</td>
<td>-</td>
<td>0.00</td>
</tr>
<tr>
<td>Second-order autocorrelation AR(2), (p-value)</td>
<td>-</td>
<td>-</td>
<td>0.98</td>
</tr>
<tr>
<td>Hansen test for overid. restrictions (p-value)</td>
<td>-</td>
<td>-</td>
<td>0.21</td>
</tr>
<tr>
<td>Number of instruments</td>
<td>286</td>
<td>286</td>
<td>286</td>
</tr>
<tr>
<td>Number of groups</td>
<td>286</td>
<td>286</td>
<td>286</td>
</tr>
<tr>
<td>Number of observations</td>
<td>2 288</td>
<td>2 288</td>
<td>2 288</td>
</tr>
</tbody>
</table>

Notes: (1) *** p<0.01, ** p<0.05, * p<0.1; Robust standard errors in parentheses
(2) The two-step system GMM estimation is performed with the *xtabond2* (Roodman 2006) command in the STATA 9.2 package. Standard errors are estimated using the Windmeijer (2005) correction.
(3) The two-step system GMM estimation is specified with the time dummies, the metropolitan dummy and the regional market size as exogenous variables (IV-style) and the remainder variables as endogenous (GMM-style).
(4) The null hypothesis in the test for first-order autocorrelation, AR(1), is autocorrelation. The null hypothesis in the test for second-order autocorrelation, AR(2), is no autocorrelation. The test statistics for AR(1) and AR(2) are satisfactory.
(5) The test statistic for the Hansen test for overidentifying restrictions (validity of the instruments) is satisfactory. The null hypothesis is not rejected. The Hansen test is robust but can be weakened by many instruments. The number of instruments is 260, which is less than the number of groups (286).
We conclude from the results in Table 3 that the two sources of persistence in start-up rates are confirmed. On top of the influence of slowly changing regional characteristics, the results suggest response mechanism from previous to current start-up rates, i.e. path-dependence in start-up activity. Both sources of persistence operate simultaneously and have individual effects. We now turn to the regional dimension in persistence.

4.2 Explaining regional persistence levels

Feedback mechanisms between current and past start-up activity in a region imply that, on top of durable and spatially sticky characteristics, there is an additional enduring advantage for regions that have shown high start-up rates in previous periods. As argued previously, this leads to the expectation that the strength of persistence in start-up rates is partly dependent on the level of start-up rates. Regions with high start-up rates over a sequence of periods are assumed to develop a strong entrepreneurial culture, or well-developed entrepreneurial capital, which induces persistence in start-up rates. The analysis so far verified the empirical relevance of slowly changing regional attributes and response mechanisms from previous to current start-up rates. We now assess the question whether persistence is stronger for regions with higher levels of start-up rates. We first present results from a transition probability analysis and then go to regression quantiles.

Figure 6 presents a graphic representation of the strength in persistence dependent on the level of start-up rates. It shows the likelihood that a region retains, over a ten-year period, its rank (defined in 10 groups of equal size) when regions are sorted according to start-up level. Thus, a high probability indicates that most regions assigned to a particular rank-group are still in that rank-group after ten years (1994-2004).

As is evident from the figure, persistence is particularly strong in the extremes of the distribution. This indicates that municipalities with either low or high levels of start-up rates are most likely to remain in the same group over the decade analyzed (1994-2004). For high levels of start-up rates this is in line with the theoretical framework outlined in the previous. However, a general problem with rank analyses is the fact that the probability of shifting groups is, by definition, lower in the extreme rank-groups, simply because members of those groups only have one other group to move into; regions in the group with the highest (lowest) start-up levels can only move downward (upward). In the middle of the distribution regions can move both upward and downward. This systemic bias may be responsible for inflating the level of persistence in the extremes. It does not undermine the whole analysis, however, because persistence in the high end of the distribution is much stronger than in the lower end. This difference cannot be the result of the bias, but it is in line with the interpretation that the existence of increasing dynamic returns makes a region less vulnerable to external shocks with persistently high start-up rates as a result.
Figure 6. 3-D plot of transition probabilities. Along the diagonal from the lower-left corner to lower right corner in the figure, the height of the figure shows the probability that a municipality in a given group will remain in the same group. Off the diagonal the figure shows that probability that a municipality in a given group will switch to another group. Group construction: all municipalities are ranked in ascending order according to start-up rate each year. Groups of municipalities are then constructed based on their year-specific rank. The principle of equal percentiles is applied in the groupings.

To substantiate the transition probability analysis, we also apply quantile regression analysis. The systemic bias related to rank analyses has no bearing on this analysis. The quantile regression technique is semi-parametric. The parameter estimates for the marginal effects of the explanatory variables are allowed to differ across the quantiles of the dependent variable. The quantile regression technique is hence allows us to test whether the estimated marginal effect of lagged start-ups rates on current start-up rates differ across the distribution of start-up rates across regions. If the relationship between current and previous start-up rates is stronger for municipalities with higher levels of start-up rates, it is supportive of our hypothesis. The estimation procedure is explained in detail in Appendix C.

We apply the quantile regression technique on the relationship between start-up rates in 1994 and 2004, i.e. a decade, across municipalities in a cross section setting. First, we estimate the relationship between start-up rates in 2004 and 1994 for each quantile without additional control variables. Second, we perform a similar estimation but include the full set of control variables.
measured in 2004, i.e. education intensity, market-size, share of services, income, employment share and metropolitan dummy. Because heteroscedastic data tend to underestimate standard errors, we apply a bootstrapped procedure.

Figures 7a and 7b presents the results for the estimation with and without control variables, respectively. In each figure, the horizontal axis measures the different quantiles and the vertical axis the magnitude of the estimated coefficients of the independent variable in focus, i.e. start-up rates in 1994. The dark area around the line represent the 5% confidence interval of the coefficient estimate.

![Figure 7a](image_url)

**Figure 7a.** Estimated marginal effect of the start-up rate in 1994 on the start-up rate in 2004 across the different quantiles of the dependent variable (start-up rate 2004). No additional control variables. Bootstrapped standard errors (3000 replications).

As illustrated by the two figures, the results of the quantile regression analysis confirms the finding in Figure 6 and support the hypothesis that the strength in persistence is related to the level of start-up activity in a region. The magnitude of the estimated coefficient for the marginal effect of start-up rates in 1994 on start-up rates in 2004 systematically increases with the quantiles. Hence, the relationship between current and previous start-up rates appear to be stronger for regions with higher levels of start-up rates. In summary, the results from both the transition probability analysis and the quantile regression analysis are consistent with the conjecture that path-dependence in start-up activity suggest a regional dimension in persistence.
Figure 7b. Estimated marginal effect of the start-up rate in 1994 on the start-up rate in 2004 across the different quantiles of the dependent variable (start-up rate 2004). Same control variables as in Table 3 (education intensity, market-size, share of services, income, employment share and metropolitan dummy). Bootstrapped standard errors (3000 replications).

5. CONCLUSIONS

Persistence is a wide-spread phenomenon in economy. Many economic indicators only change slowly over time and have a stable regional distribution. This paper ties in with general discussions about persistence by assessing persistence in a dynamic phenomenon; the start-up of new firms.

The paper distinguishes between two sources of persistence in regional start-up rates. The first that the determinants of regional start-up activity change in slow processes: As a result, start-up rates change is slow processes too and can be expected to be persistent over time. The second source is path dependence in the start-up process itself. Localized learning and demonstration effects can lead to a self-reinforcing process, which in the case of start-ups has been called an ‘entrepreneurial climate’. As argued in the paper, this suggest a response mechanisms from previous start-up activity to current ones as well as regional differences in persistence.

The empirical analyses show strong persistence over time in Swedish start-up rates. This is in line with previous results for Germany (Fritsch and Mueller 2007). By estimating a dynamic panel model with the system GMM estimator, we show that both sources of persistence are significant and operate simultaneously. In addition to regional characteristics reflecting demand- and supply-side conditions, there is a significant effect of previous start-up rates on current start-up rates. We also show that although persistence is strong in general terms, there is a regional dimension in the level of persistence. Regions with high start-up rates demonstrate higher persistence than regions with low
start-up rates. This effect is explained particularly by regional feedback mechanisms that indicate an entrepreneurial climate.

This paper has added to the empirical evidence concerning persistence in start-up rates. It has also tried to link the phenomenon of persistence in start-up rates to a more general literature on path dependence in regional economic development. In this light, persistence in dynamic phenomena, such as start-up rates, is an interesting case as it can be used as an explanation for static spatial distribution, such as the city size distribution. If a static distribution is to remain stable over time then the dynamic process feeding this distribution should follow the same distribution. Further scrutinizing persistence in dynamic processes and its drivers appears a fruitful line of research in order to further understand the different types of path dependence and their roles in economic development.

REFERENCES


Cameron, C and P. Trivedi (2009), *Microeconometrics using STATA*, STATA Press


Glaeser, E.L, J. Kolko and A. Saiz (2003), ”Consumer City”, *Journal of Economic Geography*, 1, 27-50


Appendix A

Moran’s scatterplot for start-up rates in 2004.

Figure A1. Moran’s scatterplot for start-up rates across municipalities 2004. Each element $w_{ij}$ in the applied spatial weight matrix is the inverse of the time distance by car between municipality $i$ and $j$. $w_{ij} = 0$ if the time distance between $i$ and $j$ exceeds 120 minutes.
## Appendix B

### Table B1. Static panel estimations. Estimated parameters of explanatory variables in a regression equation with start-up rates in Swedish municipalities as dependent variable.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Pooled OLS</th>
<th>FE (within)</th>
<th>FEVD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Education intensity</td>
<td>0.068***</td>
<td>0.078***</td>
<td>0.078***</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.014)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Services (share)</td>
<td>2.565***</td>
<td>1.315**</td>
<td>1.315***</td>
</tr>
<tr>
<td></td>
<td>(0.181)</td>
<td>(0.571)</td>
<td>(0.096)</td>
</tr>
<tr>
<td>Market-size</td>
<td>1.739***</td>
<td>0.728</td>
<td>4.330***</td>
</tr>
<tr>
<td></td>
<td>(0.395)</td>
<td>(0.954)</td>
<td>(0.243)</td>
</tr>
<tr>
<td>Income (log)</td>
<td>-0.304</td>
<td>-3.652***</td>
<td>-3.652***</td>
</tr>
<tr>
<td></td>
<td>(0.533)</td>
<td>(0.272)</td>
<td>(0.124)</td>
</tr>
<tr>
<td>Employment share</td>
<td>-1.673***</td>
<td>1.529</td>
<td>-0.463</td>
</tr>
<tr>
<td></td>
<td>(0.573)</td>
<td>(1.279)</td>
<td>(0.349)</td>
</tr>
<tr>
<td>Metropolitan Dummy</td>
<td>1.416***</td>
<td>-</td>
<td>1.531***</td>
</tr>
<tr>
<td></td>
<td>(0.219)</td>
<td>-</td>
<td>(0.127)</td>
</tr>
<tr>
<td>R-square</td>
<td>0.414</td>
<td>0.176</td>
<td>0.750</td>
</tr>
<tr>
<td>Number of observations</td>
<td>3 146</td>
<td>3 146</td>
<td>3 146</td>
</tr>
</tbody>
</table>

**Note:** (1)*** p<0.01, ** p<0.05, * p<0.1; Robust standard errors in parentheses
(2) FEVD refers to the fixed effects vector decomposition model developed by Plümper and Troeger (2007). It makes it possible to estimate the effect of time-invariant and almost time-invariant variables in a fixed-effect setting. In the table, income, market-size and the metropolitan dummy are specified as time-invariant.
Appendix C

In the quantile estimation technique, the parameter estimates for the marginal effects of the explanatory variables are allowed to differ across the quantiles of the dependent variable. The regression technique may be viewed as a natural extension of least squares estimation of conditional mean models, to the estimation of a group of models for conditional quantile functions. The simplest case is the median regression estimator that minimizes a sum of absolute residuals. The other conditional quantile functions are estimated by minimizing an asymmetrically weighted sum of absolute residuals. The quantile regression model specifies the conditional quantile as a linear function of covariates. For the $\theta$th quantile, a common way to write the model (Buchinsky 1998) is:

$$y_i = x_i' \beta_\theta + \varepsilon_{i\theta},$$

where $\beta_\theta$ is an unknown vector of regression parameters associated with the $\theta$th quantile, $x_i$ is a vector of independent variables, $y_i$ is the dependent variable and $\varepsilon_{i\theta}$ is an unknown error term. The $\theta$th conditional quantile of $y$ given $x$ is $Q_\theta(y|x) = x_i' \beta_\theta$ and denotes the quantile of $y_i$, conditional on the regressor vector $x_i$. The only necessary assumption concerning $\varepsilon_{i\theta}$ is $Q_\theta(\varepsilon_{i\theta}|x_i) = 0$. The $\theta$th regression quantile ($0 < \theta < 1$) of $y$ is the solution to the minimization of the sum of absolute deviations residuals

$$\min_{\beta} \frac{1}{n} \left( \sum_{\varepsilon_i > 0} |y_i - x_i' \beta| \theta + \sum_{\varepsilon_i < 0} |y_i - x_i' \beta| (1 - \theta) \right).$$

In contrast to OLS, the above equation cannot be solved explicitly since the objective function is not differentiable at the origin, but it can be solved with linear programming (Buchinsky 1998). A method of Koenker and Bassett (1982) and Rogers (1993) is generally used to estimate the variance–covariance matrix of the coefficients and generate estimates of regression coefficient standard errors. However, this method tends to underestimate standard errors for data sets with heteroscedastic error distributions (Rogers 1992). It is therefore important to use some other method for estimating standard errors, such as bootstrap re-sampling techniques. The results in Figure 7a and 7b are based on standard errors obtained by bootstrapping the entire vector of observations (Gould 1992). When the bootstrap re-sampling procedure is used, only estimates of standard error and significance levels are affected, with estimates of quantile regression coefficients remaining unchanged.