CESIS

Electronic Working Paper Series

Paper No. 18

Process Innovations in a Duopoly with Two regions[1]

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(JIBS)

September 2004

The Royal Institute of technology
Centre of Excellence for studies in Science and Innovation
www.infra.kth.se/cesis/research/workpap.htm
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[1] Status of the paper:

Process Innovations in a Duopoly with Two Region

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Abstract

Most models of duopolies with a spatial dimension refer to the 'linear' or 'circular' city. Moreover, in duopoly models with innovations, the spatial dimension is usually dropped. We bridge this gap by constructing a model with two regions, each hosting production of a differentiated quality (high and low quality). In addition, consumers are heterogenous with different willingness to pay for quality. The analysis focuses on the incentives for process innovations, which affect the unit cost of production. The model captures two common trends in the urbanization process, and their effects on the incentive for process innovations. The first is regional enlargement or reduced transport costs. We find that such changes raise the proportion of process R&D in the region producing the low-quality good relative to process R&D from the high-quality region. Second, we examine the effect on optimal process R&D of moving consumers from the low-quality to the high-quality region. This lowers the optimal amount of process R&D undertaken in the low-quality region, while it is raised in the high-quality region.

Key words: Regional agglomeration, process innovations, duopoly
JEL classification: R3, O31, L13, C72

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Preprint submitted to *25 January 2004*

1 Introduction

Most duopoly models with a spatial dimension assume a 'linear' or 'circular' city (Hotelling, 1929; Salop, 1979). On the other hand, duopoly models that focus on innovations usually exclude the spatial dimension. This paper bridges this gap by formulating a model with two regions, one supplying a high quality and the other a low quality version of a certain product. Consumers are heterogenous with different willingness to pay for quality. Within this model we change the competitive structure in two ways, reflecting two common trends in the urbanization process: regional enlargement through decreasing transport costs and agglomeration of population. The effect of these changes on the incentive for process R&D is subject to analysis, where process R&D reduces unit cost.

Duopoly models with differentiated preferences were introduced by Mussa and Rosen (1978) and extended by Shaked and Sutton (1982). Bonanno and Haworth (1998) examined the incentives for conducting process R&D and product R&D in Bertrand and Cournot duopoly models respectively, where the two firms produce high and low quality goods respectively. They found that more process innovations (process R&D) were optimally undertaken under Cournot competition than Bertrand competition. Furthermore, when comparing the behavior of the high-quality (HQ) good to that of the low-quality (LQ) good, it was found that whenever there is a difference, the HQ producer was more likely to conduct process R&D. The reverse result held for the LQ good. Rosenkranz (2003) examines the optimal mix of process and product R&D similar to the above, but also take into account strategic interaction in this choice.

In this paper, we alter the model by Bonanno and Haworth (1998) in two ways. First, the firms are located in two different regions. Second, there is a unit transport cost, which is inferred on the consumers. To make the analysis tractable, we assume that both firms have the same unit cost of production, where our predecessors instead assume a higher unit cost for the HQ good. We analyze Bertrand competition and examine the effects that an increase in distance between regions has on the incentive to conduct process R&D. We also examine what effect agglomeration size has on the incentive to innovate, by increasing the share of the population in the HQ region. The product life cycle comes in many forms. Some focus on the evolution of an entire industry (e.g. Jovanovic and MacDonald, 1994; Klepper and Simons, 2000) while others speak about the evolution of specific products (Vernon, 1966; Segerstrom et al., 1990). The latter is more common for the spatial product life cycle perspective (Norton, 1986; Klaesson, 2001). Klepper (1996) is a formalized model describing the evolution of process over product R&D, where process R&D becomes more important as the product cycle evolves. A stylized description of a product life cycle identifies four stages: 1. an entry stage where product development is strong and variety may initially increase, 2. the emergence of a dominant design (Utterback and Abernathy, 1975; Abernathy and Utterback, 1978) so that variety decreases, 3. a stage of maturity with product standardization and process routinization, 4. decline and phasing out. Much empirical support has been found for the spatial product life-cycle.[1] The theoretical model in this paper examines the effect of a population movement that makes the high-quality region larger. We ask: how does such a change affect process innovation incentives. In this way we can shed light on the importance of a large local (home) market in product life-cyle competition.

2 The baseline model

There are two regions, each with one firm producing a single variety of a good. Region H (high) produces a HQ version of the good and region L (low) a LQ version of the same good. The quality level of the LQ good is normalized to 1. Each region hosts an identical continuum of consumers each with a budget E, that can be spent consuming only one unit of the good. With no consumption the utility is E. This reflects the utility of consumption if the budget is used for a composite of goods outside the model. If used for consumption of HQ or LQ goods, the utility received is $E - p - t + \theta k$, where p is the price of the good, k its quality (k_H for the HQ good, 1 for the LQ good) and θ a taste parameter that delimits those consumers who are interested in consuming the good. The continuum of consumers are distributed uniformly over the interval $(0, 1]$. As described in Figure 1, one can identify two critical values θ^0 and θ^1

[1] For Swedish examples see Karlsson (1997); Forslund-Johansson (1997).

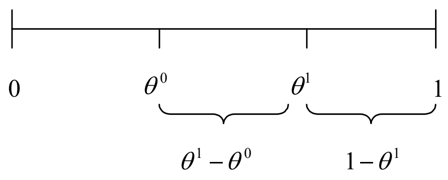

Fig. 1. The uniform distribution of consumers' preference structure.

for each region. Consider the LQ region, where θ^0 corresponds to the utility $E - p_L + \theta^0 \cdot 1$ and θ^1 corresponds to the utility $E - p_H - t + \theta^1 k_H$. The price of the LQ is p_L and p_H denotes the price of the HQ good. Values below θ^0 result in no consumption of the two competing goods and values above θ^1 result in consumption of the HQ good. For consumers with $\theta^0 < \theta < \theta^1$ we observe consumption of the LQ good. It is obvious that the consumers with low θ values are more price sensitive. Thus, a low θ reflects a higher preference for variety of goods that can be obtained from the exogenous composite good. A high θ is associated with a stronger preference for the product variants offered from the duopoly market.

Each variety of the good, in each region, has a corresponding θ for which consumers are indifferent between consuming or not. t represents the unit transport cost, which is zero if the good is produced in the home region but larger than zero if produced in the other region. Transport costs are paid by the consumers. The parameter t constitutes a major difference in comparison with models of previous authors such as Mussa and Rosen (1978), and Bonanno and Haworth (1998). Initially, we normalize the number of consumers to 1 for each region. This implies that for a value $x \in (0, 1]$ there will be $2x$ consumers of preference x or lower in each region. Later on we will modify the share of consumers to become higher in H and see what effects this agglomeration will have on the incentives to conduct process innovation. In the baseline model, demand for each good will now be derived.

2.1 Preferences and demand

We start this section by finding combinations of quality and price, and the corresponding value of θ for which consumers become indifferent between consumption of the LQ good and not consuming, and indifferent between consuming the LQ good and the HQ good. This is done for both regions and the results are used to derive demand equations for the goods. First, consider consumption of the low quality good in the LQ region. If we set the utilities equal for which the consumer is indifferent between consuming and not consuming at all, $E = E - p_L + \theta \cdot 1$ we find for region L

$$\theta_L^0 = p_L \tag{1}$$

where the subscript denotes the region. Next, we set the utilities of consumption of the LQ and HQ goods equal $E - p_L + \theta \cdot 1 = E - p_H - t + \theta k_H$, for which we find

$$\theta_L^1 = \frac{p_H - p_L + t}{k_H - 1} \tag{2}$$

The HQ good's quality is higher than that of the LQ good, which implies that $k_H > 1$. Shifting to region H, the indifference between the LQ good and no consumption is given by $E = E - p_L - t + \theta \cdot 1$ entailing

$$\theta_H^0 = (p_L + t) \tag{3}$$

The indifference between consumption of the HQ and the LQ good, $E - p_L - t + \theta \cdot 1 = E - p_H + \theta k_H$ yields the parameter

$$\theta_H^1 = \frac{p_H - p_L - t}{k_H - 1} \tag{4}$$

Finally, we need to assume $\theta^1 > \theta^0$ to obtain Figure 1. The results (1)-(4), together with the last assumption, implies

Proposition 1 $0 < \theta_L^0 < \theta_H^0 < \theta_H^1 < \theta_L^1 < 1$. *Due to transport costs, a larger share of consumers in the high-quality region choose the high-quality good, and a larger share choose the low-quality good in the low-quality region.*

PROOF. $\theta_H^0 > \theta_L^0$ follows from $t > 0$, which also implies $\theta_L^1 > \theta_H^1$. Furthermore, $\theta_H^1 > \theta_H^0$ from our assumption. There is a home-market bias in consumption of the HQ good in the HQ region if $1 - \theta_H^1 > 1 - \theta_L^1$, which follows readily from the proposition. Furthermore, the bias in the LQ-region is true if $\theta_L^1 - \theta_L^0 > \theta_H^1 - \theta_H^0$, which also follows the proposition. Transport costs explain the differences between respectively θ_L^0 and θ_H^0, and θ_L^1 and θ_H^1.

Further insight is gained by examining what happens if t rises at fixed output prices. We then find that this home bias becomes more pronounced since the share going to own production will increase. Without loss of generality, we assume that the population is equally divided between the regions. Then a change in the share going to the HQ good in H is matched by an equally large negative change of the share in the L-region:

$$\frac{d(1-\theta_H^1)}{dt} = \frac{1}{k_H - 1} = \left|\frac{d(1-\theta_L^1)}{dt}\right| \tag{5}$$

Interestingly, this symmetry is not present for the LQ-good. Here we find:

$$\begin{aligned} &\left|\frac{d(\theta_H^1 - \theta_H^0)}{dt}\right| = \left|-\frac{1}{k_H - 1} - 1\right| = \frac{k_H}{k_H - 1} > \\ &\frac{d(\theta_L^1 - \theta_L^0)}{dt} = \frac{1}{k_H - 1} \end{aligned} \tag{6}$$

This means that the decrease in the share of consumption going to the LQ good in the HQ region is not matched by an equal increase in the LQ region of its local good when t rises. Hence, total demand will drop for the LQ good. This asymmetry may seem odd at first, but remember that not all consumers choose to consume the LQ and HQ goods, unless $\theta^0 = 0$ in both regions.

Going back to Proposition 1, observe that $\theta_H^1 > \theta_H^0$ requires that

$$p_H > k_H(p_L + t) \tag{7}$$

This puts a constraint on the price level of the high quality good. Hence, in what follows we will assume that the condition in (7) applies.[2] Furthermore, $\theta_L^0 > 0$ trivially requires $p_L > 0$, and $1 > \theta_H^0$ requires

[2] This condition automatically implies, $p_H > p_L k_H - t$, which is required for $\theta_1^L > \theta_0^L$.

$$1 > p_L + t \tag{8}$$

which shows that both $t < 1$ and $p_L < 1$ since they are both positive. Observe that when t becomes too large, each region becomes a monopoly market. Turning to the demand functions, demand for the low quality good is the amount of the "number" of consumers falling within the range $\theta^0 < \theta < \theta^1$ for both regions. The interval $\theta^1 < \theta \leq 1$ contains the consumers who prefer the high quality good. This yields the demand functions D_L and D_H for each type of good respectively

$$D_H = (1 - \theta_L^1) + (1 - \theta_H^1) = 2 - 2\left(\frac{p_H - p_L}{k_H - 1}\right)$$
$$D_L = \left(\theta_L^1 - \theta_L^0\right) + \left(\theta_H^1 - \theta_H^0\right) = \frac{2\,(p_H - p_L)}{k_H - 1} - (2p_L + t)$$

2.2 Production and equilibria

The duopoly literature, frequently employs cost functions that imply constant returns to scale. While this may seem inconsistent with the existence of a duopoly structure, it may be a reasonable approximation for large production quantities. To see this we may write the total cost function as $C_i = c_0 + c_1 q_i$ where the total cost C_i is dependent on a fixed cost c_0 with the per unit cost c_1 and q_i is production. The average cost is $AC_i = \frac{c_o}{q_i} + c_1$. Hence, average cost tends towards c_1 as q_i grows larger, as in a large market. With large duopoly firms, this can be approximated by c and our constant returns function becomes:

$$C_i = cq_i$$

where c is the per unit cost (assumed to be the same for both firms). We also assume that the unit cost coefficient is low enough to ascertain that demand is positive when price equals cost, that prices and quantities are always positive. Throughout the paper we consider only the sub-game perfect equilibria where profits are maximized and non-negative. The profit conditions for the price-setting duopolists (Bertrand-game) are as follows:

$$\Pi_H = (p_H - c)\left(2 + 2\left(\frac{p_L - p_H}{k_H - 1}\right)\right)$$
$$\Pi_L = (p_L - c)\left(\frac{2p_H - 2p_L}{k_H - 1} - (2p_L + t)\right)$$

Firms choose price to maximize profits which yields

$$p_H = \frac{(k_H - 1)(4k_H - t) + 6ck_H}{8k_H - 2} \tag{9a}$$
$$p_L = \frac{c(2k_H + 1) + (1 - t)(k_H - 1)}{4k_H - 1} \tag{9b}$$

Quantities produced are

$$q_H = \frac{4c + 3t - 2 + 2k_H(2k_H - 1 - t)}{(4k_H - 1)(k_H - 1)} \tag{10a}$$
$$q_L = \frac{2\left(ck_H - c - 2ck_H^2 - 2k_H + k_H^2 + 1\right) + t(3k_H - 2k_H^2 - 2)}{(4k_H - 1)(k_H - 1)} \tag{10b}$$

Positive quantities, combined with the condition $0 < t < 1$ gives us bounds for c. From (10b) we find that $t(3k_H - 2k_H^2 - 2) < 0$ for all k_H, and hence $q_L > 0$ requires $ck_H - c - 2ck_H^2 - 2k_H + k_H^2 + 1 > 0$ for which we have[3]

$$0 < c < \frac{(k_H - 1)^2}{(2k_H^2 - k_H + 1)} \leq 1/2 \tag{11}$$

and

$$t + c < 3/2 \tag{12}$$

Together, (9) and (10) characterize a Nash equilibrium with the following profit levels

[3] $\lim\limits_{k_H \to \infty} \frac{(k_H-1)^2}{(2k_H^2-k_H+1)} = \frac{1}{2}$. $\frac{(k_H-1)^2}{(2k_H^2-k_H+1)}$ is twice continuously differentiable, with the first derivative $\frac{(k_H-1)(3k_H+1)}{(2k_H^2-k_H+1)^2} \geq 0$ and the second derivative $\frac{4(3k_H+3k_H^2-3k_H^3-1)}{(2k_H^2-k_H+1)^3} \leq 0$. Thus, $\frac{1}{2}$ is monotonically reached.

$$\pi_H = \frac{(4k_H - t - 2c)^2 (k_H - 1)}{2(4k_H - 1)^2} \tag{13a}$$

$$\pi_L = \frac{2k_H (k_H - 1)(2c + t - 1)^2}{(4k_H - 1)^2} \tag{13b}$$

Proposition 2 *The effect of an increase in transport costs is to reduce equilibrium prices and profits. Low-quality production always decreases, and high-quality production decreases if the quality difference is sufficiently large ($k_H > 3/2$). For small quality differences, high-quality production increases.*

PROOF. Examining (9) and (13), it follows directly from the assumption that $k_H > 1$. For (10) it follows for q_H that iff $t(3 - 2k_H) < 0$ or $k_H > 3/2$ the derivative is negative (conversely the derivative is positive if $k_H < 3/2$). For q_L it follows that the derivative is negative iff $3k_H - 2k_H^2 - 2 < 0$ which is true for all values of k_H.

3 Process innovations

We now introduce process innovations which aim at reductions in unit cost. This is done by a two-stage procedure analogous to d'Aspremont and Jacquemin (1988). The second stage maximizes profits conditional to already decided research investments of both firms. The first stage solves for the optimal research effort. A research investment reduces unit cost of production by r. However, we already have the result that $c \leq 1/2$. We assume that costs are constrained to be non-negative, and hence $0 \leq r_L, r_H \leq 0.5$. This puts a constraint that only interior solutions can be considered. The cost of research $c(r)$, is defined as $c(r) = \beta\frac{r^2}{2}$ with $\beta > 0$. This functional form is chosen to reflect diminishing returns of R&D. The exact value of β determines how fast diminishing returns sets in. We solve the problem by first considering the second stage, where research investments are treated as given. The profit equations are modified to

$$\tilde{\Pi}_H = (p_H - c + r_H)\left(2 + 2\left(\frac{p_L - p_H}{k_H - 1}\right)\right) - \beta\frac{r_H^2}{2}$$
$$\tilde{\Pi}_L = (p_L - c + r_L)\left(\frac{2p_H - 2p_L}{k_H - 1} - (2p_L + t)\right) - \beta\frac{r_L^2}{2}$$

Firms choose price to maximize profits which gives us the following (sub-game perfect) Nash solution for prices

$$\tilde{p}_H = \frac{t - 4k_H + 6ck_H - tk_H - 4k_H r_H - 2k_H r_L + 4k_H^2}{8k_H - 2}$$
$$\tilde{p}_L = \frac{c + t + k_H - r_H + 2ck_H - tk_H - 2k_H r_L - 1}{4k_H - 1}$$

with quantities

$$\tilde{q}_H = \frac{2c + t + 6k_H - 2r_H - 2ck_H - tk_H + 4k_H r_H - 2k_H r_L - 4k_H^2 - 2}{(4k_H - 1)(k_H - 1)} + 2$$
$$\tilde{q}_L = \frac{2k_H (2c + t + k_H - r_H - r_L - 2ck_H - tk_H + 2k_H r_L - 1)}{(4k_H - 1)(k_H - 1)}$$

and the equilibrium profit levels

$$\tilde{\pi}_H = \frac{(k_H (2c + t - 4k_H + 4) - t - 2c + 2k_H r_L - 2r_H (2k_H - 1))^2}{2(4k_H - 1)^2 (k_H - 1)} - \beta\frac{r_H^2}{2} \tag{14}$$

$$\begin{aligned}\tilde{\pi}_L = {} & ((k_H - 1)(2c + t - 1) + r_H - r_L (2k_H - 1)) \cdot \\ & (k_H (2ck_H - 4t - 3k_H - 3c + 3tk_H + 4) + c + t - 1 + r_H (3k_H - 1) - 2k_H^2 r_L) \\ & / (4k_H - 1)^2 (k_H - 1) - \beta\frac{r_L^2}{2}\end{aligned} \tag{15}$$

To obtain the optimal research levels, we maximize equations (14) and (15) above with respect to r_H and r_L. After substitution of expressions we get

$$\tilde{r}_H = \frac{2\left(2k_H-1\right)}{A} \cdot \left(2c\left(4k_H-1\right)\left(\beta+k_H(1-\beta)\right)\right.$$
$$+ 2k_H\left(k_H\left(8\beta k_H - 4k_H - 10\beta - 1\right) + 2\beta + 1\right)$$
$$\left. + t\left(k_H\left(5\beta + 6k_H - 4\beta k_H - 2\right) - \beta\right) \qquad (16)$$

$$\tilde{r}_L = \frac{1}{A} \cdot \left(c\left(4k_H-1\right)\left(\beta - 4k_H - 5\beta k_H + 8k_H^2 + 12\beta k_H^2 - 8\beta k_H^3\right)\right.$$
$$+ \beta - 28k_H - 10\beta k_H + 72k_H^2 - 64k_H^3 + 37\beta k_H^2 - 60\beta k_H^3 + 32\beta k_H^4 + 4)$$
$$+ t\left(8k_H^2 - 5k_H + 1\right) + \left(4k_H - \beta + 5\beta k_H - 4\beta k_H^2 - 2\right) \qquad (17)$$

where $A \equiv (4k_H - 1)\left(16\beta k_H - 4k_H - 4\beta - \beta^2 + 8k_H^2 - 12\beta k_H^2 + 9\beta^2 k_H - 8\beta k_H^3 - 24\beta^2 k_H^2 + 16\beta^2 k_H^3\right)$ is the common denominator. Two distinguishing features enter these optimal solutions. First, the common denominator which is helpful for analyzing the sign of the denominator in the (β, k_H)-space. Second, we can alter the values of c and t within their permissible ranges to (possibly) obtain qualitative results. We restrict the analysis to cases where $k_H \leq 5$. This may seem restrictive, but since $k_L = 1$ it implies a maximum quality level five times higher for the HQ good. This should encompass most real-world applications of differing qualities. Figure 2 and 3 shows the signs of $\tilde{r}_H$ and $\tilde{r}_L$ and the effect of increasing c and t. The mathematical results behind these graphs are derived in Appendix A. Most importantly, the grey areas show situations where the firms have incentives to to overinvest in process R&D. By this we mean that they would like to choose a higher process R&D than 0.5.[4] In addition, the areas in both figures closest to origo are negative. Since we are only interested in interior solutions with positive R&D levels, yet smaller than 0.5, the only remaining possibilities are in the "north-east areas" in Figure 2 and 3. In what follows we restrict our attention to these cases.

First we need to describe the basic elements of the two figures. The areas of positive and negative research levels are denoted by $\oplus$ and $\ominus$ signs. The two solid lines indicate where the common denominator is positive. Below the lower

[4] Since 0.5 is the maximum allowed research level, they are marked $r_H = 1/2$ and $r_L = 1/2$ respectively.

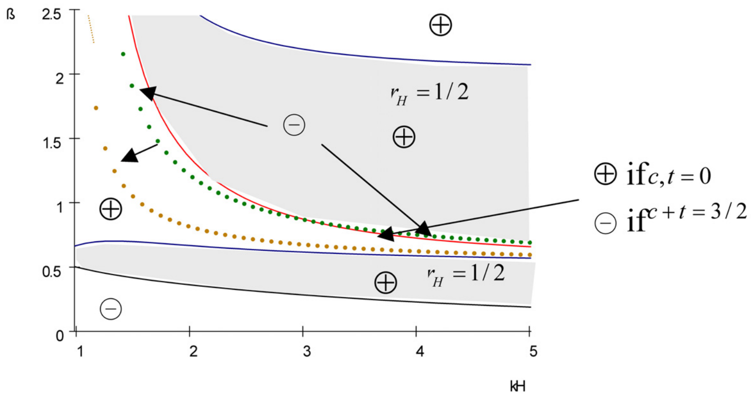

Fig. 2. Signs of $\tilde{r}_H$ in the (β, k_H)-space.

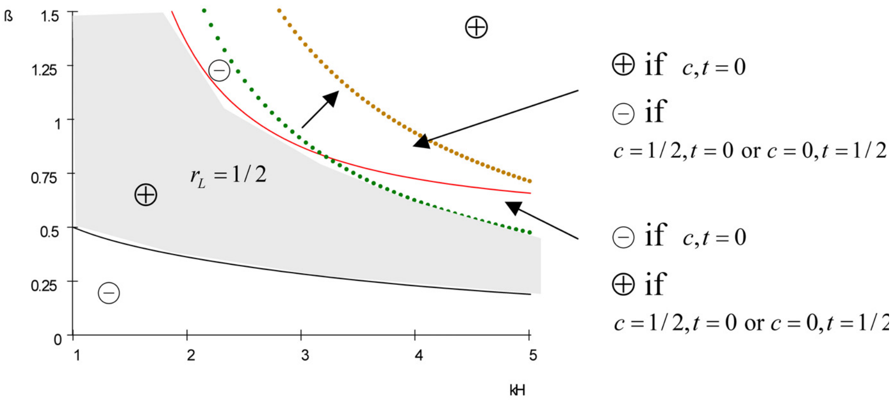

Fig. 3. More areas become positive for $\tilde{r}_L$ between the two solid lines when we increase c and/or t, but fewer areas are positive outside the second solid line.

solid line and above the top solid line the denominator is positive, between them it is negative. Above the dashed lines, the numerator is positive in both figures. For the $\tilde{r}_H$-case (Figure 2), the top dashed line shows the limiting case where c and t equal zero. The effect of increasing either of them is very similar in that they shift the dashed line toward origo, as indicated by the arrow. The lower dashed line shows the extreme where $c + t = 3/2$.

Turning to Figure 3, we find the characteristics of $\tilde{r}_L$. In contrast to the case with $\tilde{r}_H$, we see that the positive area of our interest is directly affected by the shift in the numerator, which now moves outward as we increase c or t. It seems in this case, as if there should be a negative pressure on $\tilde{r}_L$ in this case.

To obtain precise results we now differentiate our expressions (16) and (17) above with respect to t. *The opposite* of such a change can typically be interpreted as a policy measure to decrease regional distance, e.g. through infrastructure improvements.

The results are:

$$\frac{d\tilde{r}_H}{dt} = \frac{2\left(2k_H - 1\right)\left(5\beta k_H - 2k_H - \beta + 6k_H^2 - 4\beta k_H^2\right)}{A}$$

$$\frac{d\tilde{r}_L}{dt} = \frac{\left(8k_H^2 - 5k_H + 1\right)\left(4k_H - \beta + 5\beta k_H - 4\beta k_H^2 - 2\right)}{A}$$

We find that the sign of the numerator of $\frac{d\tilde{r}_H}{dt}$ is positive if

$$\begin{aligned} 5\beta k_H - 2k_H - \beta + 6k_H^2 - 4\beta k_H^2 > 0 \Leftrightarrow \\ \beta < \frac{2k_H\left(3k_H - 1\right)}{\left(k_H - 1\right)\left(4k_H - 1\right)} \end{aligned} \tag{18}$$

and for $\frac{d\tilde{r}_L}{dt}$ if

$$\begin{aligned} \left(4k_H - \beta + 5\beta k_H - 4\beta k_H^2 - 2\right) > 0 \Leftrightarrow \\ \beta < \frac{2\left(2k_H - 1\right)}{\left(k_H - 1\right)\left(4k_H - 1\right)} \end{aligned} \tag{19}$$

From this we can find the areas that positively and negatively affect $\tilde{r}_H$ and $\tilde{r}_L$ in the (β, k_H)-space. From the preceding discussion we restrict our attention to the "north-east areas" of Figure 2 and 3. Condition (18) is graphed as the dotted line in Figure 4.

Below this line the numerator is positively affected, and negatively affected above it. Combined with the denominator this means that below the dotted line but above the third solid line (from bottom to top), the effect is positive. Above the dotted line the effect is negative. Figure 5 shows the denominator together with condition (19), necessary for a positive numerator in the effect on $\tilde{r}_L$.

The result here is unambiguous: above the top solid line, our area of interest, the numerator is always negative. Since, the denominator is positive, the effect

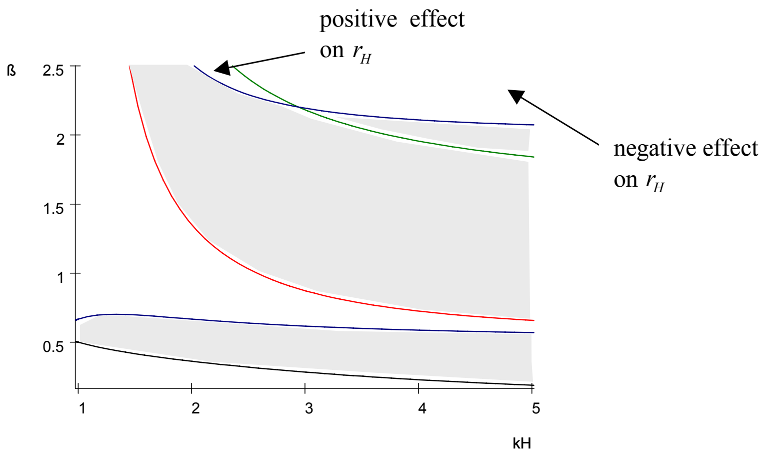

Fig. 4. An increase in t can both increase and decrease $\tilde{r}_H$.

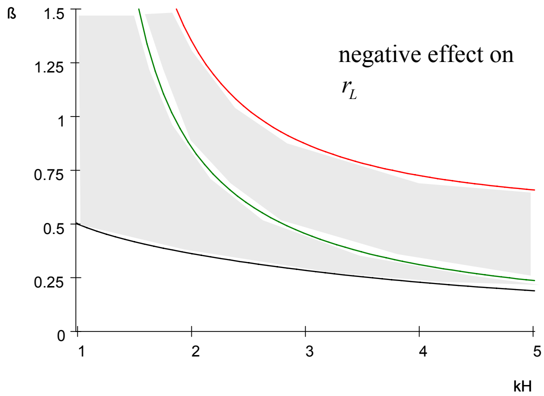

Fig. 5. $\tilde{r}_L$ is negatively affected by an increase in t.

is always negative on $\tilde{r}_L$ of an increase in t. Comparing these two results it seems as if the effect should be less negative effect for $\tilde{r}_H$ of an increase in t. We compare the numerators of the derivatives to find out. The numerator of the derivative of $\tilde{r}_H$ is bigger than that of $\tilde{r}_L$ if:

$$2\left(2k_H-1\right)\left(5\beta k_H-2k_H-\beta+6k_H^2-4\beta k_H^2\right)>$$
$$\left(8k_H^2-5k_H+1\right)\left(4k_H-\beta+5\beta k_H-4\beta k_H^2-2\right)\Leftrightarrow$$
$$\beta>\frac{2\left(2k_H-1\right)^2}{\left(4k_H-1\right)\left(8k_H^2-9k_H+3\right)}$$

This condition is plotted in Figure 6, as the bottom solid line, together with the denominator lines for reference. We see that the condition is always satisfied in our area of interest.

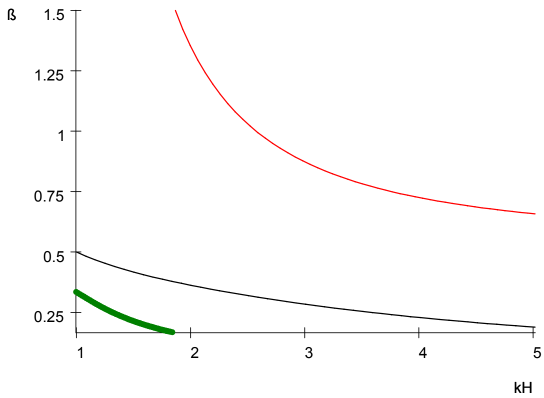

Fig. 6. The $\tilde{r}_L$-term is always more negatively affected.

Using this result, we can now summarize the effect of an increase in t on the research levels in Proposition 3.

Proposition 3 *An increase in t always lowers process R&D for the LQ good. For the HQ good process R&D may both increase and decrease. However, when they are both negatively affected, process R&D of the LQ good is always more negatively affected.*

PROOF. *The proposition follows from equations (16) and (17), the conditions (18) and (19) and the derivation of allowable regions in the (β, k_H)-space derived in Appendix A.*

4 Process innovations and population agglomeration

From a spatial product life cycle perspective, it is commonly found that goods are developed in larger, urban regions, and then, as they mature, production is typically transferred to smaller ones (Karlsson, 1999; Duranton and Puga, 2001). Hence, it should be interesting to see what effect changing sizes of regions have on the incentive for process innovation in this model. Initially we normalized population to 1 in each region. We change this assumption by letting a component $0 < \gamma < 1$ reflect the share of total population in the HQ region, which in line with product life cycle theory we assume is the central region. The total population is for comparative purposes 2 units as it was before. The analysis proceeds analogous to what we did before. That is, we first state new demand equations and then derive the sub-game perfect equilibria for prices in the second stage of the game, and the first-stage optimal process R&D levels. Then we analyze the effect of changing γ on process R&D. The changed demand equations become

$$D_H = 2(1-\gamma)(1-\theta_1^L) + 2\gamma(1-\theta_1^H) = \frac{2(2t\gamma - t + k_H - p_H + p_L - 1)}{(k_H - 1)}$$
$$D_L = 2(1-\gamma)\left(\theta_1^L - \theta_0^L\right) + 2\gamma\left(\theta_1^H - \theta_0^H\right) = \frac{2(t(1-\gamma-\gamma k_H) + p_H - k_H p_L)}{(k_H - 1)}$$

with profit equations

$$\bar{\Pi}_H = (p_H - c + r_H)\left(\frac{2(2t\gamma - t + k_H - p_H + p_L - 1)}{(k_H - 1)}\right) - \beta\frac{r_H^2}{2}$$
$$\bar{\Pi}_L = (p_L - c + r_L)\left(\frac{2(t(1-\gamma-\gamma k_H) + p_H - k_H p_L)}{(k_H - 1)}\right) - \beta\frac{r_L^2}{2}$$

Solving of the second stage Bertrand game gives us

$$\bar{p}_H = \frac{t - t\gamma - 2k_H + 3ck_H - 2tk_H + 3t\gamma k_H - 2k_H r_H - k_H r_L + 2k_H^2}{4k_H - 1}$$
$$\bar{p}_L = \frac{c + t + k_H - r_H + 2ck_H - 2t\gamma k_H - 2k_H r_L - 1}{4k_H - 1}$$

with quantities in equilibrium

$$\bar{q}_H = \frac{2(c + t - t\gamma - 2k_H - r_H - ck_H - 2tk_H + 3t\gamma k_H + 2k_H r_H - k_H r_L + 2k_H^2)}{(k_H - 1)(4k_H - 1)}$$
$$\bar{q}_L = \frac{2k_H(2c + t + k_H - r_H - r_L - 2ck_H - 2t\gamma k_H + 2k_H r_L - 1)}{(k_H - 1)(4k_H - 1)}$$

and equilibrium profit equations

$$\bar{\pi}_H =$$

$$\frac{2\left(t\gamma - t - c + 2k_H + r_H + ck_H + 2tk_H - 3t\gamma k_H - 2k_H r_H + k_H r_L - 2k_H^2\right)^2}{\left(k_H - 1\right)\left(4k_H - 1\right)^2} - \beta \frac{r_H^2}{2}$$

$$\bar{\pi}_L =$$

$$\frac{2k_H\left(r_H - t - k_H - 2c + r_L + 2ck_H + 2t\gamma k_H - 2k_H r_L + 1\right)^2}{\left(k_H - 1\right)\left(4k_H - 1\right)^2} - \beta \frac{r_L^2}{2}$$

Going back to the first stage of the game, we find after optimization of the equilibrium profit equations above, and subsequent substitution that the optimal process R&D levels are

$$\begin{aligned} \bar{r}_H = {} & \frac{4\left(2k_H - 1\right)}{B} \cdot \\ & (t\left(\beta\gamma - \beta - 4k_H + 6\beta k_H + 4\gamma k_H - 7\beta\gamma k_H + 8k_H^2 - 8\beta k_H^2 - 8\gamma k_H^2 + 12\beta\gamma k_H^2\right) \\ & + c\left(5\beta k_H + 8k_H^2 - \beta - 4k_H - 4\beta k_H^2\right) + 2\beta k_H + 4k_H^2 - 8k_H^3 - 10\beta k_H^2 + 8\beta k_H^3) \end{aligned} \tag{20}$$

$$\begin{aligned} \bar{r}_L = {} & \frac{4k_H\left(2k_H - 1\right)}{B} \cdot (\beta - 4c - 2c\beta - t\beta - 4t\gamma - 8k_H + 8ck_H - 5\beta k_H + 10c\beta k_H \\ & + 4t\beta k_H + 8t\gamma k_H + 2t\beta\gamma k_H + 4\beta k_H^2 - 8c\beta k_H^2 - 8t\beta\gamma k_H^2 + 4) \end{aligned} \tag{21}$$

with $B \equiv 4\beta + 16k_H - 28\beta k_H + \beta^2 - 64k_H^2 + 64k_H^3 + 48\beta k_H^2 - 13\beta^2 k_H + 16\beta k_H^3 - 64\beta k_H^4 + 60\beta^2 k_H^2 - 112\beta^2 k_H^3 + 64\beta^2 k_H^4$. As before, the denominator is common for both expressions. First, we examine the denominator and its sign. The denominator sign is given by the two solid lines in Figures 7 and 8. Those lines depict a very similar situation to what we found in the previous section. There is an area near origo with positive denominator, a portion in the middle with negative denominator, and outside to the "north-east" a positive denominator again. As before, we restrict attention to the "north-east" area, or to be precise, above the top denominator line. For agglomeration, γ increases, hence the derivative of $\bar{r}_H$ with respect to γ is

$$\frac{d\bar{r}_H}{d\gamma} = \frac{4t\,(2k_H - 1)\,(\beta + 4k_H - 7\beta k_H - 8k_H^2 + 12\beta k_H^2)}{B}$$

A positive effect requires (since the denominator is positive):

$$\beta + 4k_H - 7\beta k_H - 8k_H^2 + 12\beta k_H^2 > 0$$

with the solution for β :

$$\beta > \frac{4k_H - 8k_H^2}{7k_H - 12k_H^2 - 1} \tag{22}$$

The graph of this condition is plotted together with the solid-drawn denominator lines.

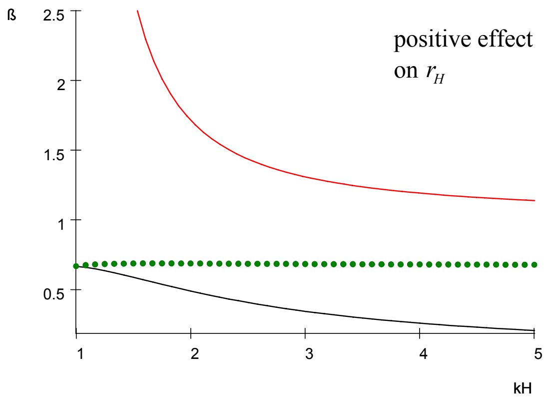

Fig. 7. Agglomeration and its effect on $\bar{r}_H$.

Since our attention is restricted to the area above the top denominator line, we are always above the dotted line. This means that the effect of a marginal positive change in γ is always positive here.

The derivative for $\bar{r}_L$ is

$$\frac{d\bar{r}_L}{d\gamma} = \frac{8tk_H\,(2k_H - 1)\,(\beta k_H + 4k_H - 2 - 4\beta k_H^2)}{B}$$

where a positive effect requires

$$\beta k_H + 4k_H - 2 - 4\beta k_H^2 > 0$$

with

$$\beta < \frac{4k_H - 2}{k_H\left(4k_H - 1\right)} \tag{23}$$

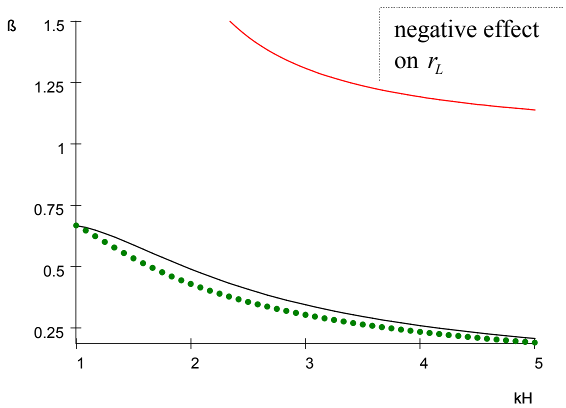

Fig. 8. Agglomeration and its effect on $\bar{r}_L$.

In this case, β is never below the dotted line, and hence the numerator is negative above the top solid line. This means that the effect on $\bar{r}_L$ is always negative of a marginal change in γ. We summarize these results as Proposition 4.

Proposition 4 *The effect of an increased concentration of consumers in the high-quality region is to increase the optimal amount of process R&D in the high-quality region and decrease the optimal amount of process R&D in the low-quality region.*

PROOF. The proposition follows from equations (20) and (21), the conditions (22) and (23) and the derivation of allowable regions in the (β, k_H)-space derived in Appendix A.

5 Discussion

We have examined what happens in terms of incentives when we increase transport costs between regions both when regional distance increases (t drops) or when people move from the LQ region. Two common tendencies in the urbanization process are that people move from peripheral (LQ) regions to central regions (HQ-region) and that regions become increasingly larger (regional enlargement). These two tendencies seem to have opposing effects on the incentive structure for process R&D. We found that increasing transport costs lowers process R&D in the LQ-region, while it may go either way for process R&D in the HQ-region. At any rate, *relative process R&D* drops for the LQ-region. Turning the argument the other way around, improvement of infrastructure or decreased transport costs, raises this relative process R&D for the LQ-region. Lower transport costs makes it more profitable for companies in peripheral regions to try to break into the market of the HQ-region. They do so by becoming more competitive through lowering of unit costs, which occur as a consequence of process R&D. Second, movement of people from the LQ-region to the HQ-region unanimously lowers the willingness to do process R&D in the LQ-region while it rises for the HQ-region. Thus, it seems that being closer to a large market makes it more profitable to enter there, and to take market shares through lowering process R&D. Policy measures in this model, aimed at changing transport costs or closeness to markets for producers, have repercussions on process R&D and national system of innovation.

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A Process R&D: permissible values in (β, k_H)-space

To find permissible ranges of β, k_H we analyze the numerators and the common denominator. We start off with the denominator. We find two solutions for $A = 0$:

$$\beta_A = \frac{-8k_H + 6k_H^2 + 4k_H^3}{9k_H - 24k_H^2 + 16k_H^3 - 1} \pm \frac{C + 2}{9k_H - 24k_H^2 + 16k_H^3 - 1}$$

where $C = 2\sqrt{-9k_H + 33k_H^2 - 62k_H^3 + 57k_H^4 - 20k_H^5 + 4k_H^6 + 1}$. The solutions are valid since $k_H \neq 1$ and $k_H \neq \frac{1}{4}$ and are plotted in Figure A.1 below (and in Figures 2 and 3).

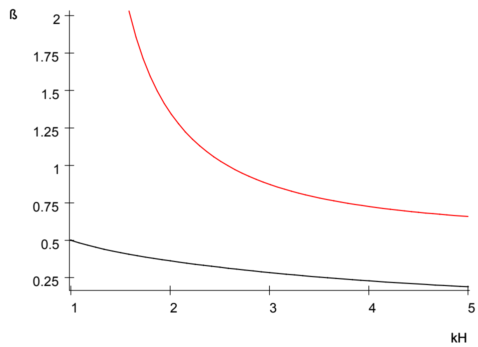

Fig. A.1. Denominator conditions: regional enlargement.

Below the bottom line the denominator is positive, between them negative and above the top line it is positive again.

A.1 Characterization of $\tilde{r}_H$

Turning to the numerator of $\tilde{r}_H$, we look at its values when $t = 0$, $c = 0$ and $t = 1$, $c = 1/2$. Intermediate values were tested and provide intermediate results. When $t = 0$, $c = 0$ we find that

$$\tilde{r}_H = \frac{4k_H\left(2k_H - 1\right)\left(2\beta - k_H - 10\beta k_H - 4k_H^2 + 8\beta k_H^2 + 1\right)}{A}$$

The denominator is the same. The numerator is positive when

$$\beta > \frac{\left(k_H + 4k_H^2 - 1\right)}{2\left(k_H - 1\right)\left(4k_H - 1\right)} \tag{A.1}$$

In the case where $t = 1$, $c = 1/2$, $\tilde{r}_H$ becomes

$$\tilde{r}_H = \frac{2\left(2k_H - 1\right)\left(14\beta k_H - k_H - 2\beta + 8k_H^2 - 8k_H^3 - 28\beta k_H^2 + 16\beta k_H^3\right)}{A}$$

where the numerator is positive whenever

$$\beta > \frac{k_H\left(8k_H^2 - 8k_H + 1\right)}{2\left(k_H - 1\right)\left(2k_H - 1\right)\left(4k_H - 1\right)} \tag{A.2}$$

Figure A.2 shows these numerator conditions together with the denominator lines.

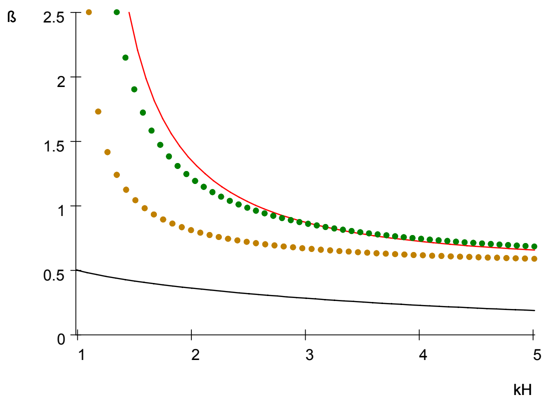

Fig. A.2. Numerator and denominator lines for $\tilde{r}_H$.

The top dotted line shows condition A.1 and the lower dotted line condition A.2. Figure A.2 shows that the area below the bottom solid line is negative, above the it but below a dotted line it will be positive. Above a dotted line but below the top solid line it is negative and above the top solid line it is positive. Lastly, we examine through a combination of numeric analysis and analytical solutions the conditions for overinvestment, or $\tilde{r}_H > 0.5$. From numerical analysis we find that the least restrictive situations arise in the case where we let $t = 1$, $c = 1/2$. Hence setting

$$\frac{4\beta + 2k_H - 36\beta k_H - 20k_H^2 + 48k_H^3 - 32k_H^4 + 112\beta k_H^2 - 144\beta k_H^3 + 64\beta k_H^4}{A} = \frac{1}{2}$$

The solutions are

$$\beta\left(\tilde{r}_H = \frac{1}{2}\right) = \frac{2\left(37k_H^2 - 10k_H - 62k_H^3 + 40k_H^4 \pm D + 1\right)}{\left(4k_H - 1\right)^3\left(k_H - 1\right)}$$

where $D \equiv \sqrt{170k_H^2 - 20k_H - 796k_H^3 + 2225k_H^4 - 3772k_H^5 + 3796k_H^6 - 2144k_H^7 + 576k_H^8 + 1}$. These solution are graphed in Figure 2 of the main text. Letting $t = 0$, $c = 0$, we will have larger areas where overinvestment occurs.

A.2 Characterization of $\tilde{r}_L$

For $\tilde{r}_L$ we examined the case $t = 0$, $c = 0$ as well. However, $t = 1$, $c = 1/2$ had no solution and we therefore examined intermediate situations such as $t = 1/2$ and $c = 0$, $t = 0$ and $c = 1/2$, and $t = 1/2$, $c = 1/2$. We found that the general tendency of the numerator line was very similar when we raised c and/or t. We show here the cases $t = 0, c = 0$ and $t = 0$, $c = 1/2$. First, $t = 0, c = 0$ gives us

$$\tilde{r}_L = \frac{\left(8k_H^2 - 5k_H + 1\right)\left(\beta - 8k_H - 5\beta k_H + 4\beta k_H^2 + 4\right)}{A}$$

For a positive numerator we require:

$$\beta > \frac{4\left(2k_H - 1\right)}{\left(4k_H - 1\right)\left(k_H - 1\right)} \tag{A.3}$$

In the case where $t = 0$, $c = 1/2$ we find

$$\tilde{r}_L = \frac{(1/2)\,(2k_H - 1)\,(36k_H - \beta + 9\beta k_H - 48k_H^2 - 24\beta k_H^2 + 16\beta k_H^3 - 8)}{A}$$

with the condition for β:

$$\beta > \frac{6\,(2k_H - 1)}{(k_H - 1)\,(4k_H - 1)} \tag{A.4}$$

Condition A.3 and A.4 are plotted together with the denominator in Figure A.3

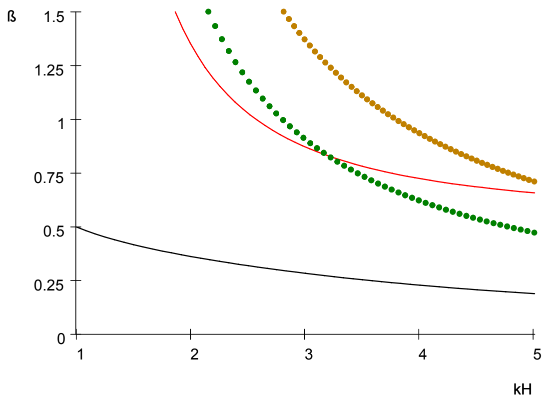

Fig. A.3. Numerator and denominator lines for $\tilde{r}_L$.

The dotted line moves outward as we increase c and/or t. Below the bottom solid line, $\tilde{r}_L$ is negative. For the largest part between the solid line it is positive. The other parts are best seen in Figure 3 in the main text. The analysis of overinvestment of $\tilde{r}_L$ was done mainly by numerical analysis. The shaded area of Figure 3 always yield overinvestment and hence is not allowed.