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Macroeconomic Factors and Swedish Small and Medium-Sized Manufacturing Firm Failure

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Macroeconomic Factors and Swedish Small and Medium-sized Manufacturing Firm Failure

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Abstract

This paper employs a time series cointegration approach to evaluate the relationship between manufacturing firm failure and macroeconomic factors for the Swedish manufacturing sector in the period 1986 – 2006. It uses quarterly data for this period. We found that in long run a firms' failure is negatively related to the level of industrial activity, money supply, GNP and economic openness rate, and positively related to the real wage. Time series Error Correction Model (ECM) estimates suggest that macroeconomic risk factors impinge on firm failures on the same direction in both the short run and the long run and that adjustment to stabilise the relationship is quite slow.

Keywords: estimating companies' failure, macroeconomic factors, cointegration analysis, diagnostic tests

1. Introduction

The attention given to the firms' failure phenomenon is not new. Several theoretical and empirical studies have been dedicated to this problem in recent decades. Why this renewal of interest? The recent boost given to this topic has its origins in the multiplication of the involuntary loss of businesses, as well as its dramatic consequences, notably for the receivers and their different stakeholders (creditors, employees, etc).

The increased mobility of capital, skills and entrepreneurship, now at the heart of the process of globalisation, has become even more important. Increasing technological achievements, the adoption of investment liberalisation policies by many countries, privatisation and firms' switch in emphasis to geographical diversification are some of the more important explanations for the strong expansion in structural change recorded in the past decades. The question in this situation concerns the expansion of an economic policy that is necessary to increase the industrial sector's adaptability so as to meet the demand for structural change.

In previous studies, it was noted that manufacturing firms are more likely to fail than service or trading businesses. Among firms, those with traditional technology are most likely to go bankrupt and the firms that went bankrupt had fewer employees on average than all the business sector companies; they were also less likely to be exporters (Sharabany, 2004).

The failure rate of firms is determined by three factors: 1) The effectiveness of the management and adequacy of its capital. Young firms are more likely to fail than experienced firms (Altman 1993). Small firms are more prone to bankruptcy because their access to the credit markets is more limited than that of large firms (Bernanke and Gertler 1995). 2) A

shock to a specific industry, such as its exposure to import reform, tariff reform, etc., and 3. Risk deriving from macroeconomic or monetary factors. 3) The last factor has been less investigated. In order to eliminate possible negative development in different industry sectors, different macroeconomic factors' impact on business failure should be tested.

The main purpose of the paper is to investigate whether major macroeconomic can explain manufacturing firms' failure factors in Sweden. A general Error Correction Model (ECM), according to Engle and Granger's (1987) methodology, is conducted for this purpose. The specification of the model is then evaluated by means of well-behaved statistical diagnostic tests.

The econometric results show the key relationships between firms' bankruptcies; the majority of potential determinants are in accordance with the theory. Different specifications have been tested, including different combinations of explanatory variables and possible lags. In order to find the best specification we also kept the conformity between the expected signs for coefficients and those obtained as a criterion. Indeed, a reduction in GNP and the economic openness rate should *entail an increase in bankruptcies.* Therefore, the signs attached to these variables should be negative. Whereas an increase in new incorporations or wages, the birth of new companies and real wages, should result in *an increase in bankruptcies*, it should be expected that the signs attached to these three variables are positive.

The paper is organized as follows: Section 2 presents a review of some relevant literature. In Section 3 we discuss the methodology used in this paper. Section 4 shows the time series properties of the data. Section 5 presents the analyses of the empirical results, followed by the discussion and conclusion in section 6.

2. Review of the literature

The early literature represents the aggregate probability of bankruptcy. It depends on four typical macroeconomic variables - the real wages rate, the real interest rate, real output, and actual and expected inflation, as well as two typical balance sheet variables: real liabilities and real net worth. It must be stressed, however, that in the latter case it is the distribution of the variables that really matters.

Several previous studies have employed econometric specifications inspired by the variables mention above. Altman (1983) considers the macroeconomic determinants of corporate failure. He makes use of first-differenced distributed-lag models to show that business failure is negatively related to aggregate activity (measured by GNP), money market conditions (M2) and investor expectations. Platt and Platt (1994) extended the analysis for the U.S. by means of a cross-section correlated autoregressive model. They provide evidence for a negative relationship between business failure and economic activity (employment and corporation) while costs (real wages and business formation rate) enter with a positive indication. A list of papers with a focus on short run dynamic relationships includes Wadhwani (1986), Hudson (1986), Young 1995 and Cuthbertson and Hudson (1996). They all employed time series techniques to find the aggregate economic activity business failure rate, with the expected results. Vlieghe (2001) and Liu (2004) explicitly addressed the issue of disentangling the short run and long run behaviour of corporate failure relating to macroeconomic factors, by the means of time-series cointegration and error correction models. A long run relationship emerges between the business failure rates on the one hand, and economic activity, corporate lending and the real interest rate on the other. Vlieghe (2001) found that the debt/GDP ratio, the real interest rate, deviation of GDP from trend, and the real wage are long run determinants of the liquidation rate. The birth of new companies, index of property prices,

and the nominal interest rate has a significant short run effect. Finally, Fabling and Grimes (2005) used regional data for New Zealand to assess the scope of regional variation in insolvency using a SUR model.[1]

As a subject, the failure of manufacturing firms has not traditionally received much attention from macroeconomic theory; the main academic work on the subject has focused on potential lenders' concerns with an individual firm's creditworthiness. Yet casual observation suggests that macroeconomic developments may have a significant role to play in explaining firm failure: movements in the aggregate failure rate have coincided with changes in macroeconomic performance. It seems worth providing additional evidence about the relationship between business failure and macroeconomic risk factors for a country like Sweden.

3. Methodology

In this section we discuss the research design and the model specification together with the selected variables.

3.1 Research design

The majority of work on firm failure has been done in order to analyse the determinants of failure. As we are also interested in why firms fail, we used cointegration analysis and error correction model (ECM) for this purpose.

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[1] Geroski and Gregg (1997), and Lennox (1999) examine the role of the economic cycle and find proxy macroeconomic effects significant here; an improvement in the macroeconomic environment reduces the likelihood of failure, by means of probability linear model.

In this paper we used quarterly data about the manufacturing sector for 1986-2006 in order to empirically assess the nature and properties of bankruptcy transmission and mechanisms in Sweden. A possible way of distinguishing between the two alternative hypotheses consists of quantitatively disentangling the short and long run co-movement between sources of systematic (i.e. non-diversifiable) risk for the total business failure rate. Non-stationary time series econometrics proves to be particularly helpful in accomplishing this task.

Moving from an emphasis on a simple theoretical framework to a framework that highlights the key relationships between the business bankruptcy rate and macroeconomic conditions, the empirical analysis consists of three steps. First, time series root tests are used to assess the presence of stochastic non-stationary variables in a set of variables comprising manufacturing firms failure rate, by the level of industrial activity measured by the value added manufacturing sector, real wage, birth rate of new firms, growth rate of money supply and economic openness measured by the exports. Second, the existence of a cointegration relationship between firms' failure and other integrated aggregate variables is investigated. The long-term relationship in terms of a cointegration vector is determined by referring to several alternative estimators. Third, the results from the previous step are used in estimating a time series ECM relationship. The last step allows us to distinguish between short-term and long-term influences and compare them.

Numerous test statistics and diagnostic checks have been used as tools in our strategy for selecting models. In this paper we also provide a strategy for selecting the appropriate model by examining the adequacy of the different models through utilising several misspecification tests. These tests are: the Breusch (1978) and Godfrey (1978) common tests for autocorrelation; the White (1980) test for heteroscedasticity; the Engle (1981) LM test for

Autoregressive conditional heteroscedasticity ARCH; the Ramsey (1969) RESET test for omitted variables and functional misspecification; the Jarque and Bera (1987) test for non-normality; and finally, the cumulative sum of squares (CUSUMSQ) test for parameter stability, due to Brown, Durbin, and Evans (1975). Some brief illustrations of these tests are included in the Appendix and further details can be found in the above-mentioned references. By following such a strategy, one can avoid inadequate models that could lead to extremely misleading results and inferences.

The main purpose of the paper was to investigate whether major macroeconomic factors in Sweden can explain the manufacturing firm failure by using a cointegration analysis and an Error Correction Model (ECM), according to Engle and Granger's (1987) methodology. These models are selected by means of reliable statistical diagnostic tests.

We present two main results. First, we found acceptable evidence for a cointegration relationship between the firm failure rate, industrial activity, economic openness and money supply (with negative indicators) and the real wage and rate of new birth firms (with positive indicators). The aggregate economic activity was measured by GNP (with negative indicators).

It was noted that manufacturing companies are more likely to fail than service or trading companies. Among manufacturing companies, those with traditional technology are the most likely to go bankrupt. Also, on average the companies that went bankrupt had fewer employees than did all business-sector companies, and they were less likely to be exporters.

3.2. The model's variables

In general, the macroeconomics factors used to identify which companies are most at risk in the previous studies were wages, nominal and real interest rate, GDP and both actual and expected inflation. However, in our study we focus on macroeconomic variables such as the level of industrial activity, aggregate economic activity, real wage, the birth rate of new firms, the growth rate of money supply and economic openness.

Lags of bankruptcies (i.e., lagged dependent variable) are included in the model as independent variables for two reasons. Firstly, these variables are important, and their appearance will become evident in recession ensuring they can be assumed to be closely related. Secondly, to capture the effect of possible missing variables that might not be included here. The data shows strong variation due to economic performance in the manufacturing industry. The data also reveals an increase in bankruptcies in the 1980s, 1990s and in 2000. These were the years that the Swedish economy was in structural stagnation. On the other hand, the period between 1984 and 1990 shows the reverse picture. In that period the economy was undergoing great expansion, declining bankruptcies and increasing inflation.

The manufacturing sector's added value is a measure of the sum of incomes (including profits) paid to the production factors used by the industry in order to produce its commodities. Thus, it is a measure of the level of industrial production. When they looked at the birth rate of new companies Chaueau and Madjaqui (1988) observed an association between the change in firm failures and the change in firm population. Moreover, the frequency distribution of failures, with respect to age of the firm, showing that one-half of all failures occur within a firms' first five years and almost one-third within the first three years. The increased fragility of some young firms is mainly due to insufficient equity capital,

inadequate managerial abilities, commercial constraints and the time required to capture its part of market. We would expect that the failure rate increases when the new birth rate is important (see Altman 1983).

Economic openness is an important variable for a small open economy, such as Sweden, with a high dependency on trade. The increase in the economic openness rate is entirely attributable to the development of international trade. For some the exchange rate with foreign markets tends to encourage the birth of businesses, while allowing them to increase the outlets for national production and hence economy of scales. Otherwise, for others, opening the market increases the degree of the competition in exposed sectors. Competition over prices, quantities and quality becomes tough and difficult to supervise. However, we cannot ignore the stimulating effect of this competition. It pushes the businesses to tackle international competition. The primary impact of market openness on growth is expected to be positive. In fact, it is connected to the spread of technological progress that covers various channels, such as the import of more modern goods and the stimulation of research and development, etc.

To take into account the effect of the outside exchange's development on bankruptcies' evolution, we suggest export activity. Therefore, we expect a positive relationship between the development of international trade and the growth of business, hence there is a negative link between the evolution of bankruptcies and the evolution of the economic openness rate. The next independent variable is the cost of a workforce unit. We found strong evidence for a cointegration relationship between bankruptcies and the cost of a workforce unit (with positive indicators)

Altman (1983) has underlined the relevance of variations in price level as the fifth factor with a small link to the failure phenomenon. In fact, inflation can have a short-term impact upon a firms' propensity to survive, i.e. increases in prices, especially unanticipated increases, tend to be inversely correlated to the failure rate. The next independent variable is the money supply's growth rate. For example, a more restrictive domestic monetary policy implies that bank loans will be more expensive, both for businesses and households. There will be a decrease in demand for commodities, the request for new capacity declines and so do bankruptcies. Finally, GNP was included in this model as an independent variable. The result of estimates for this variable was significant (with negative indicators).

3.3. Model specification

This paper will only focus on the major macroeconomic factors in order to obtain a satisfactory explanation for their effects on bankruptcy. The general linear model of firm failure is estimated and tested. Since some of the independent variables have been shown not to be statistically significant (namely exports, the interest rate and exchange rate), we only include the variables that have been shown to be significant in our model specification and in the results thereafter. Note that all variables are in logarithmic form and the GNP, VA, HW and INV are measured in billions of Swedish crowns. In order to estimate the long term relationship between the number of failed companies in Sweden and these variables we created the following model:

$$\ln COMF_t = \beta_0 + \beta_1 \ln (GNP)_t + \beta_2 \ln (VA)_t + \beta_3 \ln (RW)_t + \beta_4 \ln (NF)_t +$$

$$\beta_5 \ln (econop)_t + \beta_6 \ln (MS)_t + \theta_i \ln (COMF)_{t-i} + u_t. \tag{1}$$

where:

COMF   = number of companies failure in Sweden

GNP   = aggregate economic activity (recession, measured by the

                   gross national product)

VA   = level of industrial activity (measured by the manufacturing sector value added

RW   = real wage

NF   = birth rate of new firms (measured by the change of

                   investment expenditures)

Econop   = economic openness

MS   = growth rate of money supply

$COMF_{t-i}$ = the lagged numbers of companies failure in Sweden as

                   proxy variable for recession phases in Sweden

β_i and θ_i = parameters associated with the independent and lagged

                   dependent variables, respectively (to be estimated)

u   = error terms assumed to be; $u_t \sim N(0, \sigma^2)$.

3.4. The Error Correction Model (ECM)

To capture both the long term and the short term dynamics of company failure in Sweden, we created an error correction model using the Engle-Granger (1987) methodology. This involved the inclusion of the lagged residuals from the regression in model (1) in a new model, with all the involved variables taken in the first difference. The ECM regression will take the following form:

$$\Delta(COMF)_t = \beta_0 + \beta_1 \; \Delta(GNP)_t + \beta_2 \; \Delta(VA)_t + \beta_3 \; (RW)_t + \beta_4 \; \Delta(NF)_t + \beta_5 \; \Delta(\; EConop \;)_t +$$

$$\beta_6 \; \Delta(MS)_t + \theta_i \; \Delta(COMF)_{t-i} + \alpha \; u_{t-1} + e_t. \qquad\qquad (2)$$

Where, Δ is the first difference and α is the error correction parameter (often referred to as the speed of adjustment parameter).

4. Results

In this section we present the most important results of the study. First, model (1) has been created using the Ordinary Least Squares (OLS) method with all included variables at levels. This equation represents the long term relationship between the number of failed companies in Sweden and the included independent variables mentioned above. After testing for their stationarity, the residuals from this model are then used in the ECM model (2) to capture the long term and the short term dynamics of company failure in Sweden. We present the results of these two equations in the following Tables 1 and 2, respectively.[2]

Table 1 about here.

The results from calculating the long term cointegration relationship between company failure in Sweden and the other included variables reveal that an increase in the real wage or the rate of new births should result in an increase in bankruptcies. We therefore expect that the indicators attached to these two variables are positive. Whereas a reduction in the level of the rate of new births, money supply, the industrial activity variable, economic openness rate and

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[2] Different specifications have been tested, including different combinations of explanatory variables and possible lags. To find the best specification we also kept the conformity between the expected indications of coefficients and those obtained as a criterion.

aggregate activity should entail an increase in bankruptcies. Therefore, the signs attached to these variables should be negative. Three lags of the dependent variable (also as proxy for recession) have been shown to be significant and hence are included in the analysis. The values for the associated parameters are small in magnitude and have positive indicators.

The second stage of the Engle-Granger methodology suggests the calculation of an error correction model. This has been done by including the lagged residuals from model (1) in model (2). Model (2) states that the first difference of the dependent variable depends on the first differences of the other independent and lagged dependent variables and also on the equilibrium error term u_{t-1}. If the latter differs from zero, then the model is out of equilibrium.

Table 2 about here.

The coefficient of the lagged residuals has shown to be a negative indicator with a value around unity (see Table 2 below). According to the Engle-Granger theorem, a negative coefficient is to be expected if there is a cointegration relationship between the variables. The absolute value of α in model (2) decides how quickly equilibrium is restored. The results from this model also reveal that Δ aggregate activity (measured by GNP) has the expected major impact and negative indicators, while the Δ (industrial activity) variable has negative indicators but is not significant.

The estimated parameters for Δ (real wage) and Δ (rate of new birth firms) are small and also have positive indicators, implying a small effect on the increment of the number of failing companies. Note that these parameters are only significant at the 7% level. For Δ (economic openness) and Δ (growth rate of money supply), aggregate activity, the estimated parameters

have negative indicators. This means that when these factors increase, the number of companies failing will decrease. The calculated parameter for Δ (economic openness) is not statistically significant. Three lags of the dependent variable have also shown to be significant with positive indicators. Results from several diagnostic tests have shown that models (1) and (2) are well specified. These results are presented in the following Table 3 and the two graphs for the CUSUMSQ test.

Table 3 and CUSUMSQ test about here.

5. Conclusion

This study has attempted to shed some light on the influence of macroeconomic variables on the failure of small and medium sized Swedish businesses during the period 1986-2006 using quarterly data. On this basis, cointegration analysis and error correction methodology (ECM) were applied in order to calculate the short term and long term relationship between bankruptcies and macroeconomics variables. The application of cointegration analysis is computationally demanding and requires a background of statistical theory in time series models. Nevertheless, some important results can be obtained in a straightforward manner with use of the appropriate statistical package. According to this, we found that it is possible to explain the varying bankruptcy rate in Sweden by trends in manufacturing output, real wages, and the birth rate of new companies, money supply, recession (GNP) and economic openness.

To the best of our knowledge, our results regarding the impact of macroeconomic instability on bankruptcies showed a long term relationship between the bankruptcy rate and economic

activities - they are more likely to go bankrupt during unstable years. Acquisition activity is also subdued in these years. Additionally, there are fewer bankruptcies and more acquisition during an economic upturn, particularly when measured against the Swedish business cycle. The finding of contemporaneous increases in bankruptcies and the decline in acquisition activities in a period of instability or low economic growth suggests the need for further work on assessing causal relationships between the two processes.

In addition, evidence is found for effects of real wages and the birth rate of new companies during Sweden's transition to a steady state. We interpret these results as that fiscal and monetary policy arrangements would affect business bankruptcies differently in the short and the long term. The short term policies (i.e. demand side policies) are likely to have less influence than policies aimed at helping to moderate labour, fixed costs and interest payments.

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Table 1. Parameter Estimates for Model (1).

Variables	Coefficients	Standard errors	P-values
Constant	24.82	5.16	0.00
GNP	- 2.63	0.41	0.00
Industrial activity	-0.69	0.42	0.10
Real wage	0.46	0.23	0.05
Birth rate of new companies	0.28	0.09	0.00
Economic openness	-3.09	0.77	0.00
MS	-5.05	0.72	0.00
$COMF_{t-1}$	0.31	0.07	0.00
$COMF_{t-2}$	0.19	0.08	0.02
$COMF_{t-3}$	0.26	0.07	0.00

$R^2 = .94$

Table 2. Parameter Calculations for the ECM Model (2)[*]

|	Coefficients	Standard errors	P-values
Constant	0.01	0.02	0.68
Δ (GNP)	-2.29	0.60	0.00
Δ (industrial activity)	-0.40	0.44	0.37
Δ (Real wage)	0.36	0.20	0.07
Δ (rate of new birth)	0.28	0.16	0.07
Δ (econimic openness)	-1.55	1.23	0.21
Δ (money supply)	-6.50	2.81	0.02
$\Delta(COMF_{t-1})$	0.39	0.11	0.00
$\Delta(COMF_{t-2})$	0.23	0.09	0.01
$\Delta(COMF_{t-3})$	0.36	0.08	0.00
u_{t-1}	-1.03	0.15	0.00

- $R^2 = .94$, All variables in model (1) are included in the first difference form in model (2) and all the
  results are presented, even if some of the parameters are not significant.

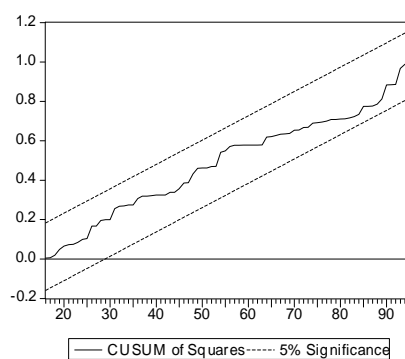Table 3: P-values of Misspecification Tests for the two models.

Test	Cointegration regression	Error correction Model
Autocorrelation (1 lag)	.781	.471
Autocorrelation (4 lags)	.788	.500
Heteroscedasticity	.319	.946
ARCH	.661	.761
RESET	.878	.300
Non-normality	.447	.749

CUSUMSQ test:

Cointegration regression   Error correction Model

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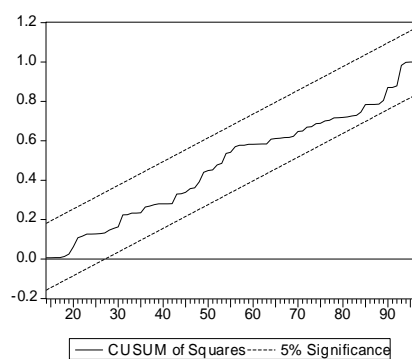
Appendix;

Diagnostic Tests

1. The CUSUMSQ test

This test is used for time series and checks for structural changes. In the CUSUMSQ test Recursive Residuals (RR) calculated by the Kalman Filter are used. We now describe the construction of recursive residuals and the Kalman filter technique. The recursive residuals can be computed by forward or backward recursion. Only forward recursion is described, backward recursion being analogous.

Given N observations, consider the following linear model with the corresponding vector of coefficient b expressed as b_t, implying that the coefficients may vary over time t. The hypothesis to be tested is $b_1 = b_2 = , \ldots, = b_N = b$. The OLS estimator based on N observations is:

$$b = (X'X)^{-1} X'y ,$$

where X is a N by k matrix of observations on the regressors, and y is an N by 1 vector of observations for the dependent variable. Suppose that only r observations are used to estimate b. Then for $r > k$, where k is the number of independent variables,

$$b_r = (X_r'X_r)^{-1} X_r'y_r , \quad r = k+1, \ldots, N .$$

Using b_r, one may "forecast" y_r at sample point r, corresponding to the vector X_r of the explanatory variables at that point.

Recursive residuals are now derived by estimating equation (2 . 2 . 1) recursively in the same manner, that is by using the first k observations to get an initial estimate of b, and then gradually enlarging the sample, adding one observation at a time and re-estimating b at each step. In this way, it is possible to get (N-k) estimates of the vector b, and correspondingly (N-k-1) forecast errors of the type:

$$W_r = y_r - X_r b_{r-1} , \quad r = k+1, \ldots, N ,$$

where b_{r-1} is an estimate of b based on the first $r - 1$ observations. It can be shown that, under the null hypothesis, these forecast errors have mean zero and variance $s^2 d_r^2$, where d_r is a scalar function of the explanatory variables, equal to $[1 + X_r'(X'_{r-1}X_{r-1})^{-1} X_r]^{1/2}$.

Then the quantity:
$$W_r = \frac{y_r - X_r b_{r-1}}{[1 + X_r'(X'_{r-1}X_{r-1})X_r]^{1/2}} , \qquad r = k+1, \ldots, N ,$$

gives a set of standardized prediction errors, called "recursive residuals". The recursive residuals are independently and normally distributed with mean zero and constant variance s^2. As a result of a change in the structure over time, these recursive residuals will no longer have zero mean, and the CUSUM and CUSUMSQ of these residuals can be used to test for structural change.

CUSUM involves the plot of the quantity:

$$V_r = \sum_{t=k+1}^{r} W_t / \sigma^*, \qquad\qquad r = k+1, \ldots, N,$$

where s^* is the estimated standard deviation based on the full sample.

CUSUMSQ is based on the square of the recursive residuals, and involves the plot of the quantity:

$$S_r = \sum_{t=k+1}^{r} W_t^2 / \sum_{t=k+1}^{N} W_t^2 , \qquad\qquad r = k+1, \ldots, N.$$

Note that the quantity S_r lies between zero and one ($S_r = 0$ if $r < k+1$ and $S_r = 1$ if $r = N$), and the expected value of S_r is $E(S_r) = (r - k) / (N - k)$. A significance test can be performed by drawing a pair of lines parallel to the mean value line. The reference lines take the form: $[(r - k) / (N - k)] \pm C$. The required values for C, corresponding to specific values for a (the significance level), can be found in Johnston (1984, Table B-8, p. 560). If the plot of the CUSUMSQ crosses the confidence bounds, then the hypothesis of no structural change is rejected (assuming that the model specification is correct).

The role of the two tests is somewhat different, with the CUSUMSQ test more oriented toward random parameter variation than the CUSUM test (Martin and Porter, 1985). We will use the CUSUMSQ in our analyses, as this is reported to be more powerful than the CUSUM test (cf. Garbade, 1977, p.57).

2. The Breusch-Godfrey-test

The Breusch-Godfrey test can be separated into several stages:

1. Run an OLS on the following "theoretical" model:

$$y_t = \alpha + \beta X_t + \theta_i y_{t-i} + \varepsilon_t$$

This gives us $\hat{\varepsilon}_t$

2. Run an OLS on:

$$\hat{\varepsilon}_t = \alpha + \beta X_t + \theta_i y_{t-i} + \rho_1 \hat{\varepsilon}_{t-1} + \rho_2 \hat{\varepsilon}_{t-2} + ... + \rho_P \hat{\varepsilon}_{t-P} + u_t$$

This equation can be used for any AR(P) process. From this equation the unrestricted residual sum of squares (RSS_u).

The restricted residual sum of squares (RSS_R) is given from the following equation:

$$\hat{\varepsilon}_t = \alpha + \beta X_t + \theta y_{t-1} + v_t$$

The null hypothesis is:

$$H_0 : \rho_1 = \rho_2 = ... = \rho_P = 0$$

3. Calculate an F-test:

F=((RSS_R-RSS_U)/p) / (RSS_U/(T-k-P))

This has a distribution: F(P,T-k-P) under the null hypothesis.

The Breusch-Godfrey test can be tested for AR(P) processes which gives this test a clear advantage over other available tests for autocorrelation.

3. The Ramsey RESET-test

RESET test stands for Regression Specification Error Test. The test is very general and can only tell you if you have a problem or not. It tests for omitted variables and incorrect functional forms or misspecified dynamics and also if there is a correlation between the error term and the independent variable. The null hypothesis is:

H_0: E $(\varepsilon_i/X_i) = 0$

H_1: E $(\varepsilon_i/X_i) \neq 0$ (and an omitted variable effect is present)

Thus, by rejecting the null hypothesis indicates some type of misspecification. First a linear regression is specified:

$$y_i = \alpha + \beta X_i + \varepsilon_i$$

This gives the restricted residual sum of squares (RSSR). After the RSS_R has been found the unrestricted model is presented by adding variables (three fitted values):

$$y_t = \alpha + \beta X_i + \theta_1 \hat{y}_i^2 + \theta_2 \hat{y}_i^3 + u_t$$

This gives us the unrestricted residual sum of squares (RSS_U). In the third step the RESET-test uses a F-test:

F= ((RSS_R - RSS_U)/number of restrictions under H_0) / (RSS_U / (N- number of parameters in unrestricted model))

The F-test checks if $\theta_1 = \theta_2 = 0$, if $\theta_1 = \theta_2 \neq 0$ we have an omitted variable or a misspecification in the model.

4. The White's test

This test is a general test where we do not need to make any specific assumptions regarding the nature of the heteroscedasticity, whether it is increasing, decreasing etc. The test only tells us if we have an indication of heteroscedasticity.

$$H_0 : \sigma_i^2 = \sigma^2 \qquad\qquad \forall i$$

The alternative hypothesis is not H_0, anything other than H_0.

The test can be divided into several steps:

1. Run an OLS on:

$$y_i = \alpha + \beta_1 X_{1i} + ... + \beta_k X_{ki} + \varepsilon_i$$

From this equation we get $\hat{\varepsilon}_i$ which is used as a proxy for the variance.

2. Run an OLS on:

$$\hat{\varepsilon}_i^2 = \alpha_0 + \alpha_1 X_{1i} + ... + \alpha_k X_{ki} + \alpha_{k+1} X_{1i}^2 + ... + \alpha_{k+k} X_{k+k}^2 + \alpha_{k+k+1} X_{1i} X_k + \delta_i$$

Where k is the number of parameters. The variance is considered to be a linear function of a number of independent variables, their quadratic and cross products. Thus, the X:s is used as a proxy for Z.

3. Calculate an F-test:

Restricted model: $\hat{\varepsilon}_i^2 = \alpha_0' + \delta_i'$

From this test the restricted residual sum of squares (RSS$_R$) is measured.

The F-test is:

F=((RSS$_R$-RSS$_U$)/k) / (RSS$_U$/(n-k-1))

Where $H_0 : \alpha_i = 0 \quad \forall i = 1,2...k$

5. The Engel's LM test for ARCH effects

This is a test for AutoRegressive Conditional Heteroscedasticity (ARCH). The ARCH process can be modeled as:

$$y_t = \alpha + \beta X_t + \varepsilon_t$$

where the Variance of ε_t conditioned on ε_{t-i} : $\text{Var}(\varepsilon_t \backslash \varepsilon_{t-i}) = \alpha_0 + \alpha_1 \varepsilon_{t-i}^2$

1) Run OLS on the original model and get: $\hat{\varepsilon}_t$. Square it and use it in the folloing unrestricted model:

2) $$\hat{\varepsilon}_t^2 = \alpha_0 + \alpha_i \hat{\varepsilon}_{t-i}^2 + \delta_t$$

3) Test whether $\alpha_i = 0$, for any i = 1, 2 , . . . By an F-test as before.

6. The Jarque-Bera test For Non-Normality

The test for non-normality is normally done before one test for heteroskedasticity and structural changes.

The test used here for testing for normal distribution is the Jarque-Bera test. The Jarque-Bera test is structured as follows:

$$T \left[1/6\hat{b}_1^2 + 1/24(\hat{b}_2 - 3)^2 \right]$$
$$b_1 = \mu_3 /(\mu_2)^{3/2}$$
$$b_2 = \mu_4 /(\mu_2)^2$$

Where T is the total number of observations, b_1 is a measure for skewness and b_2 is a measure for kurtosis. The μ are different moments. The test has a chi-square distribution with two degrees of freedom under the null hypothesis of normal distribution. The two degrees of freedom comes from having one for skewness and one for kurtosis.