CESIS Electronic Working Paper Series

Paper No. 214

The Properties of Procedures Dealing with Uncertainty about Intercept and Deterministic Trend in Unit Root Testing

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February 2010

The Royal Institute of Technology
Centre of Excellence for Science and Innovation Studies (CESIS)
http://www.cesis.se

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Abstract

The classic Dickey-Fuller unit-root test can be applied using three different equations, depending upon the inclusion of a constant and/or a time trend in the regression equation. This paper investigates the size and power properties of a unit-root testing strategy outlined in Enders (2004), which allows for repeated testing of the unit root with the three equations depending on the significance of various parameters in the equations. This strategy is similar to strategies suggested by others for unit root testing. Our Monte Carlo simulation experiments show that serious mass significance problems prevail when using the strategy suggested by Enders. Excluding the possibility of unrealistic outcomes and using *a priori* information on whether there is a trend in the underlying time series, as suggested by Elder and Kennedy (2001), reduces the mass significance problem for the unit root test and improves power for that test. Subsequent testing for whether a trend exists is seriously affected by testing for the unit root first, however.

Key words: Unit Roots, Deterministic Components, Model Selection

Running title: Unit Root Testing with Equation Uncertainty

JEL classification: C30

Introduction

Unit root testing is one of the most common procedures in modern time series analysis. This has arisen since determination of unit root status is a prerequisite to figure out whether correlation between variables in a regression is spurious or whether cointegration exists.[1] The earliest and one of the simplest unit root tests used is the Dickey and Fuller (1979) test, which is based on one of the three regression equations below

$$\Delta y_t = by_{t-1} + \varepsilon_t \tag{1}$$

$$\Delta y_t = a + by_{t-1} + \varepsilon_t \tag{2}$$

$$\Delta y_t = a + ct + by_{t-1} + \varepsilon_t \tag{3}$$

where y_t is the variable being tested for unit root, t is time, ε_t is white noise, a, b, and c are parametric constants, and the first difference operator is represented by Δ.[2] The null hypothesis of unit root in this formulation is expressed as a zero restriction on b. Note that equations (1) and (2) are simply restricted forms of equation (3). The unit root test is one-sided and the distribution of the test statistic is non-standard under the null, and differs depending upon which equation is used. Dickey and Fuller (1979) have provided the special critical values for a finite set of observations[3] and MacKinnon (1991) has offered a means for determining the special critical values more generally.

Unfortunately, one often does not know which of the three equations is appropriate for testing. Some authors have recommended sequential testing strategies to determine unit roots under such circumstances. The time series econometrics textbook by Enders (2004) for example presents a sequential testing strategy, which is repeated in the applied econometrics textbook by Asteriou and Hall (2007). Some authors, including Enders (2004), have recommended sequential testing strategies to determine unit roots under such circumstances. These strategies typically start with testing the unit root using equation (3), and depending

[1] The issue of spurious regression was illustrated by Granger and Newbold (1974) as their simulation showed that a regression of variables with unit roots produced significant correlation even if the variables are independent. This point was also proved analytically by Phillips (1986). The issue of cointegraton was brought up by Granger (1981) and tests for cointegration were developed by Engle and Granger (1987), Phillips (1987), Johansson (1988), and Johansson and Juselius (1990), among others.

[2] According to Said and Dickey (1984), the test equation should be augmented with lags of Δy_t if autocorrelation exists for the error term ε_t.

[3] Dickey and Fuller (1981) show that the distribution of the F test is nonstandard when a unit root is included in the null hypothesis. They provide new critical values for the F test under such circumstances.

upon the result of that test and others, allow consideration of testing using the more restricted equations (1) and (2). Such techniques are likely to suffer from the problem of mass significance due to the repeated testing.

The purpose of this paper is to evaluate the precision of inference based on the sequential testing techniques of Enders (2004) and show the effect of *a priori* elimination of possible outcomes based upon arguments by Elder and Kennedy in their 2001 article "Testing for Unit Roots: What Should Students Be Taught?". To achieve this purpose, we conduct Monte Carlo experiments on the sequential testing technique put forward by Enders (2004), and the unit root testing strategy suggested by Elder and Kennedy (2001).

The rest of this paper is organised in the following way. Next section describes the sequential testing strategy outlined by Enders (2004) and evaluates this strategy. Section 3 presents the unit root testing strategy with prior restrictions as suggested by Elder and Kennedy (2001), and this is also evaluated. The last section concludes the paper.

I. The Sequential Unit Root Testing Strategy of Enders (2004)

Of the three previous equations, equation (3) is the most general with equations (1) and (2) nested in it. Since each of these three equations has one of two unit root statuses—a unit root exist or it does not—we can consider six possible models as shown in Table 1. These models are presented based on different restrictions imposed on the parameters of the underlying data generating process. The main goal is to find out whether y has a unit root or not. To achieve this it is often crucial to appropriately include or not include the intercept and/or time trend term in the unit-root test equation.

To deal with the lack of information of whether an intercept or time trend should be included, Enders (2004) provides a multiple-step sequential strategy for going about the testing for unit roots. He attributes this methodology as being a modification of one suggested by Dolado, Jenkinson, and Sosvilla-Rivero (1990). Elder and Kennedy (2001) list other sources with similar recommendations: Perron (1988), Holden and Perman (1994), and Ayat and Burridge (2000). The Enders strategy is shown in Figure 1 for a Dickey-Fuller (DF) environment.[4]

[4] Formally, Enders presented his strategy in an augmented DF environment with various lags of Δy as additional explanatory variables to handle possible autocorrelation in the error terms, but this modification is suppressed in this paper for simplicity. The later simulations will have no inherent autocorrelation in the error

Table 1. Definitions of Models Based on the General Equation $\Delta y_t = a + by_{t-1} + ct + \varepsilon_t$

Model (1)	Model (2)	Model (3)	Model (4)	Model (5)	Model (6)
$a = 0$ $b = 0$ $c = 0$	$a = 0$ $b < 0$ $c = 0$	$a \neq 0$ $b = 0$ $c = 0$	$a \neq 0$ $b < 0$ $c = 0$	$a \neq 0$ $b = 0$ $c \neq 0$	$a \neq 0$ $b < 0$ $c \neq 0$
Unit root, no intercept, no time trend	Stationary around zero equilibrium	Unit root with drift	Stationary around non-zero constant equilibrium	Unit root with intercept and time trend	Deterministic trend, "trend stationary"

Enders does not specify the concluding model as is done in Figure 1. He just provides the conclusion of whether there is a unit root or not. The concluding model noted in the figure is the current authors' interpretation of the implied model. The following is also done in this paper to complete the model-selection interpretation. Toward the top of the figure one conclusion is "Decide no unit root (model (2), (4) or (6))". In that case, which of these three models is ultimately concluded is determined by standard *t*-statistic testing. If $c = 0$ can be rejected, then we conclude model (6); if it cannot be rejected then we estimate $\Delta y_t = by_{t-1} + a + \varepsilon_t$ and test whether $a = 0$ can be rejected, with an affirmative indicating model (4) and a negative answer indicating model (2). Likewise, further down near the middle of the figure one conclusion is "Decide no unit root ((model (2) or (4))". In that case, which of the two models is concluded is determined by standard *t*-statistic testing: if $a = 0$ can be rejected, then we conclude model (4); if it cannot be rejected then we conclude model (2).

terms, so this seems reasonable. Also, to be fair to Enders, he warns that "no procedure can be expected to work well if it used in a completely mechanical fashion. Plotting the data is usually an important indicator of the presence of deterministic regressors." (p. 214)

Figure 1. Enders Strategy

Estimate $\Delta y_t = a + by_{t-1} + ct + \varepsilon_t$

↓

Can b = 0 be rejected using DF critical values?→ Yes → Decide no unit root (model (2), (4) or (6))

↓ No

Can b = 0 and c = 0 be rejected using DF critical values?

No ↓ Yes

Can b = 0 be rejected using a normal distribution?

↓ No

↓Yes

Decide unit root (model 5) Decide no unit root (model 6)

Estimate $\Delta y_t = a + by_{t-1} + \varepsilon_t$

↓

Can b = 0 be rejected using DF critical values? → Yes → Decide no unit root ((model (2) or (4))

↓ No

Can a = 0 and b = 0 be rejected using DF critical values? → Yes

No ↓

Can b = 0 be rejected using a normal distribution?

↓ No ↓ Yes

Decide there is a unit root (model 3) Decide no unit root (model 4)

Estimate $\Delta y_t = by_{t-1} + \varepsilon_t$

↓

Can b = 0 be rejected using DF critical values? → Yes → Decide has no unit root (model (2))

↓ No

Decide there is a unit root (model (1))

Notes: DF stands for Dickey-Fuller. Each question about parameters is answered based on the last estimated equation before the question.

To make an evaluation of the Enders (2004) strategy, we conduct some Monte Carlo simulations using a program developed for GAUSS. The design of these simulations is as follows. Fifty observations are generated according to parameters consistent with models (1), (3), or (5), i.e. the models with a unit root, using an error term drawn independently from a standard normal distribution. The Enders strategy with the model-choice extensions noted previously is then employed to determine whether a unit root exists or not and the implied model given the results. This experiment is performed 5000 times and the percent of times the null hypothesis of a unit root is rejected is reported in Table 2 and the percent of times each model is chosen is reported in Table 3. The nominal significance level indicated (10%, 5%, or 1%) is applied on every hypothesis test performed.

Table 2. Frequency of Rejecting Unit Root When There Is a Unit Root Based on the General Equation $\Delta y_t = a + by_{t-1} + ct + \varepsilon_t$; Using Enders Strategy

Nominal Significance Level	a= 0, b = 0, c = 0: true model is Model (1)	a = 0.25, b = 0, c = 0: true model is Model (3)	a = 1, b = 0, c = 0: true model is Model (3)	a = 1, b = 0, c =0.4: true model is Model (5)
10%	23.0%	15.3%	10.5%	0.1%
5%	11.5%	7.6%	5.1%	0.0%
1%	2.6%	2.9%	1.2%	0.0%

Table 3. Percentage Choosing Various Models, Based on the General Equation

$$\Delta y_t = a + by_{t-1} + ct + \varepsilon_t$$

3(i). Model Chosen, given true model is (1); $a = 0$, $b = 0$, $c = 0$

Nominal Significance Level	Model (1)	Model (2)	Model (3)	Model (4)	Model (5)	Model (6)
10%	73.5 %	8.7 %	1.7 %	14.1 %	1.8 %	0.3 %
5%	86.1 %	4.9 %	1.3 %	6.8 %	1.1 %	0.0 %
1%	97.0 %	0.9 %	0.2 %	1.7%	0.2 %	0.0 %

3(ii). Model Chosen, given true model is (3); $a = 0.25$, $b = 0$, $c = 0$

Nominal Significance Level	Model (1)	Model (2)	Model (3)	Model (4)	Model (5)	Model (6)
10%	63.4 %	1.8 %	19.5 %	11.1 %	1.8 %	2.4 %
5%	76.8 %	1.2 %	14.4 %	5.7 %	1.2 %	0.7 %
1%	92.7 %	0.4 %	5.1 %	1.5 %	0.4 %	0.1 %

3(iii). Model Chosen, given true model is (3); $a = 1$, $b = 0$, $c = 0$

Nominal Significance Level	Model (1)	Model (2)	Model (3)	Model (4)	Model (5)	Model (6)
10%	0.0 %	0.0 %	87.7 %	0.4 %	1.8 %	10.1 %
5%	0.0 %	0.0 %	93.7 %	0.3 %	1.2 %	4.9 %
1%	0.0 %	0.0 %	98.6 %	0.1 %	0.2 %	1.2 %

3(iv). Model Chosen, given true model is (5); $a = 1$, $b = 0$, $c = 0.4$

Nominal Significance Level	Model (1)	Model (2)	Model (3)	Model (4)	Model (5)	Model (6)
10%	0.0 %	0.0 %	0.0 %	0.0 %	99.9 %	0.1 %
5%	0.0 %	0.0 %	0.0 %	0.0 %	100.0 %	0.0 %
1%	0.0 %	0.0 %	0.0 %	0.0 %	100.0 %	0.0 %

The results of the simulation experiments, presented in Tables 2 and 3, may be interpreted as follows:

- With no intercept and no time trend, the frequency of concluding a unit root when there is actually a unit root is too low (actual size is too high relative to nominal size). This ensues because the Enders methodology allows for rejection of the null hypothesis of the unit root at various steps, so mass significance becomes very problematic. As Table 3 indicates, model choices in this situation tend to be spread over all possible models, with model (4) as the main alternative followed by model (2).
- Without a time trend but with a weak drift term, there is still over-rejection of the null hypothesis of a unit root, although not as much as when there is no drift term.
- Without a time trend but with a strong drift term, there are less tests that are relevant in the Enders methodology—testing model (1) versus model (2) is not done since a = 0 is always rejected. This is visible in Table 3 also, with models (1) and (2) never chosen when $a = 1$, $b = 0$, and $c = 0$. As a result, the mass significance problem is reduced, and the actual sizes are closer to the nominal sizes, although the actual sizes are too high. Model (6) is the main incorrectly chosen alternative, and the percentage choice of that model closely matches the nominal sizes.
- With a time trend and a unit root, the size suddenly becomes too low, almost always failing to reject the null hypothesis of a unit root. This comes about because this simple Dickey-Fuller test is not sufficient for testing unit roots when there is both a unit root and a time trend. An additional regressor such as t^2 would be needed to test for a unit root under such circumstances since the number of deterministic regressors needs to be at least as numerous as the number deterministic components (Harris and Sollis (2003), p. 45). [5]

[5] Strategy S1 in Ayat and Burridge (2002) includes a t^2 regressor in the first unit-root test of a strategy similar to that of Enders, but at the cost of more mass significance difficulties and too-frequent spurious identification of a quadratic trend when a strong linear trend exists.

II. Unit Root Testing Strategy with Prior Limitations

Elder and Kennedy (2001) have criticized methods like Enders' method based on the following arguments:

- they are double testing and triple testing for unit roots (the mass significance problem),
- they allow for unrealistic outcomes, and
- they do not take advantage of prior knowledge of time series growth.

The problem of mass significance has already been demonstrated by the simulation results that we have just presented. Cutting down on possible models based on removing unrealistic outcomes and using prior knowledge about time series growth provides a way to deal with the mass significance problem. Elder and Kennedy (2001) claim that model (5) should not be allowed due to its explosive nature,[6] and model (2) should not be allowed since a stationary process around an equilibrium of exactly zero is unlikely. When there is no prior knowledge about growth in the variable, only models (1), (3), (4), and (6) should be allowed. However, if we have a good reason to think there is a time trend or a trend created by a drift term we can narrow down our choices further to models (3) and (6) only. If instead we have a good reason to think there is no time trend or a trend created by a drift term, we can narrow down our choices further to models (1) and (4). How the Enders strategy is modified by Elder and Kennedy's suggestions (referred to as the Elder and Kennedy strategy) when there is no prior knowledge of the variable's growth is listed below.[7]

<u>Elder and Kennedy Strategy, No Prior Knowledge of Growth in Variable</u>

A. Estimate the equation $\Delta y_t = a + by_{t-1} + ct + \varepsilon_t$, and test whether $b = 0$ can be rejected using DF critical values. If it can be rejected, conclude no unit root, and if not, conclude there is a unit root.[8]

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[6] By explosive is meant the series has a rate of change that is ever increasing or ever decreasing. Elder and Kennedy (2001) more widely criticize this model as unrealistic for economic time series, with explosiveness being *one* reason why it is unrealistic. Elder and Kennedy refer to Perron (1988) and Holden and Perman (1994) on discussing the issue of the unrealistic nature of this model.

[7] Again, the presentation is in a Dickey-Fuller environment rather than an augmented Dickey-Fuller one, matching the presentation in Elder and Kennedy.

[8] Elder and Kennedy accept that some double testing for unit roots could be appropriate at this point to improve power. The problem of mass significance is reintroduced with that double testing, however.

B. If b = 0 can be rejected in step A, use standard t testing to determine whether c = 0 can be rejected (i.e. conclude model 6) or not (i.e. conclude model 4).

C. If b = 0 cannot be rejected in step A, estimate the equation $\Delta y_t = a + \varepsilon_t$, and test whether a = 0 can be rejected using standard t-statistic testing. If it can be rejected, conclude model (3), if not, conclude model (1).

If nonzero growth in the y variable is known *a priori*, the Elder and Kennedy strategy becomes the same as step A above, with the conclusion of no unit root implying model (6) and the conclusion of unit root implying model (3) (neither step B nor step C need be done). If zero growth in the y variable is known *a priori*, the Elder and Kennedy strategy becomes the same as step A above with ct excluded in estimation. The conclusion of no unit root would then imply model (4) and the conclusion of a unit root implying model (1) (neither step B nor step C need be done).

The issue of what constitutes appropriate prior knowledge may not be clear however. Some variables for theoretical reasons have growth or not, and that certainly constitutes prior knowledge. However, if the growth (or not) of a variable is determined prior to testing solely by looking at the data or previous similar data, then that "eyeball test" arguably should be considered part of the testing procedure and again could lead to a mass significance problem after being followed by other tests.

Examining the Elder and Kennedy strategy with no prior knowledge of growth, we can see that they avoid the issue of mass significance on the unit root test by avoiding multiple testing of the unit root; determination of whether a unit root or not exists is based entirely on one test. A second test after that is suggested by Elder and Kennedy only to determine whether growth or not is involved along with the stationarity or nonstationarity determined by the unit root test. Due to this structure in their strategy, they completely control for size in their unit root test – the actual size for that test should be very close to the nominal size, and simulations we have done have indicated that is true.[9]

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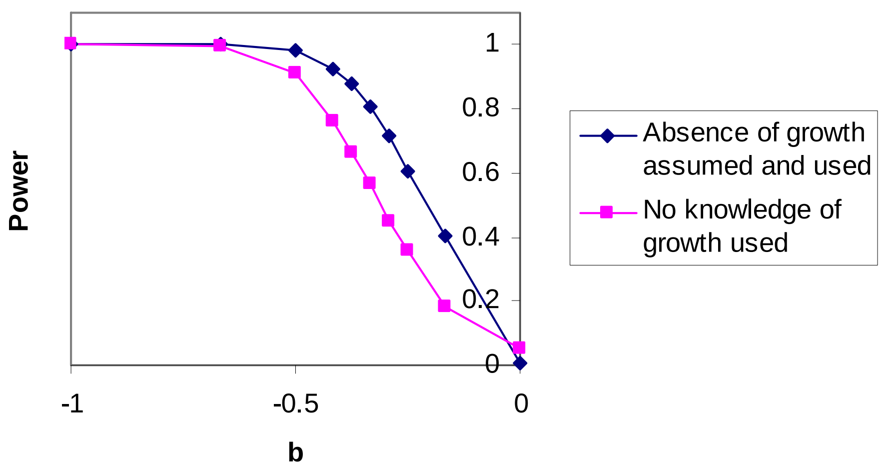
[9] Another strategy with good size properties under similar conditions is strategy S3 of Ayat and Burridge (2000), which includes pre-testing for the linear trend using Vogelsang's (1998) t-PS1 statistic, which is invariant to the unit root, followed by a single unit-root test appropriate given the results of the trend test. That

The power of the unit root test of course depends on the true parameter values and the associated issue of which alternative situation—stationary around a nonzero constant or trend stationary—is the true one, and the associated true parameter values. If stationarity around a nonzero constant (model 4) is the true model, then the power function has the typical shape, with small magnitudes of b resulting in power close to the size, and successively larger magnitudes (more negative) of b resulting in successively higher power. This is demonstrated in the simulation results of Figure 2 when no prior knowledge of growth is used and when correct prior knowledge that there is no growth is used. The figure also shows that power improvement is possible from utilizing prior correct knowledge about non-growth when stationarity around a nonzero constant is true, confirming the statement on this matter by Elder and Kennedy.

Figure 2. Power Function When DGP is $\Delta y_t = by_{t-1} + 1 + \varepsilon_t$ Using Elder and Kennedy Strategy with No Prior Knowledge of Growth and 5% Nominal Size for All Testing.

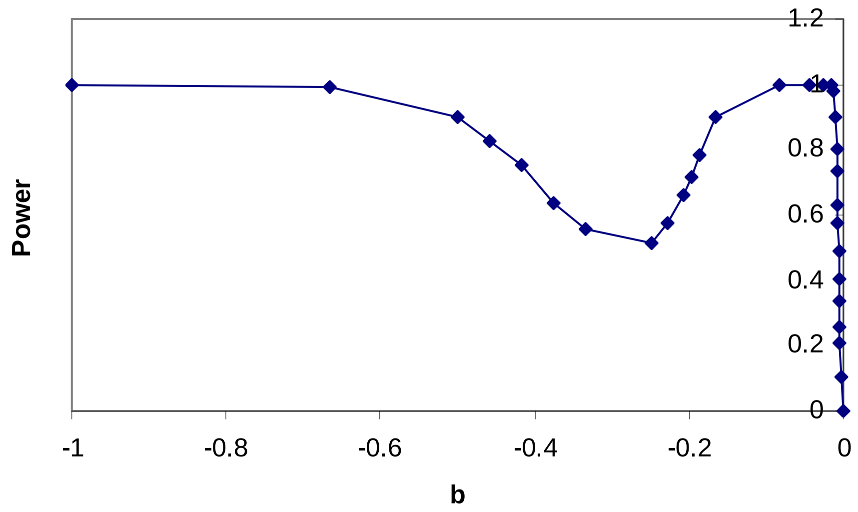


If trend stationarity (model 6) is the true model, the power function representing the likelihood of accepting stationarity correctly does not differ whether or not we use correct *a priori* information on growth in the Elder and Kennedy method. This is true since the equation estimated for the unit root test would be the same regardless of the *a priori* information; the estimated equation would include time as an explanatory variable along with a constant regardless. However, the power function under such circumstances can have an

strategy was found to have good size properties for the unit root test, but the power properties for that test were not impressive.

unusual shape (rising, falling, and rising again with higher magnitudes of b) as demonstrated in Figure 3. This is perhaps attributable to the fact that for values of b near 0, there is slow convergence so the variable can take on attributes that seem like those of a variable generated by the excluded model (model (5)), in which there is nonstationarity around a trend leading to explosiveness.

Figure 3. Power Function When DGP is $\Delta y_t = 1 + by_{t-1} + t + \varepsilon_t$ Using Elder and Kennedy Strategy with No Prior Knowledge of Growth and 5% Nominal Size for Testing.



The Elder and Kennedy strategy using no prior knowledge on growth is admirable in its control of size on the unit root test. However, those authors also suggest a second test (step B or C) as a possibility for those interested in what is the appropriate model to conclude upon. At this point the legitimacy of the size of the second test becomes questionable due to the prior testing for the unit root.

In Table 5 we present for various data generating processes (DGP) the simulated size or power on the first test and second test in the Elder and Kennedy strategy when there is no prior knowledge of growth in the variable. The first test is the unit root test. The second test is either the test for a drift if a unit root is not rejected in the first test, or the test for a deterministic time trend if a unit root is rejected in the first test. The table also shows the frequency of choosing the correct DGP structure (unit root no drift, unit root with drift, stationary around nonzero constant, or trend stationary). Each cell in the table presents three numbers. The first number in each case is the simulated value when a nominal size of 10% is used on all tests. The second number is the corresponding value when a nominal size of 5% is used on all tests, and the third number is the corresponding value when a nominal size of 1% is used on all tests.

Table 5. Size and Power Properties for the First and Second Tests of the Elder and Kennedy Testing Strategy with No Prior Knowledge of Growth

True data generating process, with $\varepsilon_t \sim N(0,1)$	Size 1st test	Power 1st test	Size 2nd test	Power 2nd test	Frequency Choosing true DGP structure
(i) $\Delta y_t = \varepsilon_t$	9.8% 4.9% 0.8%	-	19.9% 9.9% 1.7%	-	72.3% 85.7% 97.5%
(ii) $\Delta y_t = 1 + \varepsilon_t$	9.6% 4.5% 1.0%	-	-	100.0% 100.0% 100.0%	90.4% 95.5% 99.0%
(iii) $\Delta y_t = 0.25 + \varepsilon_t$	10.3% 5.1% 0.9%	-	-	68.2% 53.7% 26.6%	61.2% 51.0% 26.4%
(iv) $\Delta y_t = -0.5 y_{t-1} + 1 + \varepsilon_t$	-	96.9% 90.3% 62.7%	0% 0% 0%	-	96.9% 90.3% 62.7%
(v) $\Delta y_t = -0.05 y_{t-1} + 2 + \varepsilon_t$	-	11.4% 6.8% 1.7%	56.1% 41.2% 17.6%	-	5.0% 4.0% 1.4%
(vi) $\Delta y_t = -0.5 y_{t-1} + 1 + 0.4t + \varepsilon_t$	-	96.4% 89.9% 60.3%	-	100.0% 100.0% 100.0%	96.4% 89.9% 60.3%
(vii) $\Delta y_t = -0.1 y_{t-1} + 1 + 0.4t + \varepsilon_t$	-	47.4% 32.5% 12.1%	-	100.0% 100.0% 100.0%	47.4% 32.5% 12.1%
(viii) $\Delta y_t = -0.5 y_{t-1} + 1 + 0.2t + \varepsilon_t$	-	96.6% 90.1% 60.5%	-	97.7% 63.9% 4.3%	94.4% 57.6% 2.6%
(ix) $\Delta y_t = -0.5 y_{t-1} + 1 + 0.1t + \varepsilon_t$	-	96.9% 90.4% 62.7%	-	1.4% 0.0% 0.0%	1.4% 0.0% 0.0%

Notes:

a. The first test is the unit root test. The second test is either the test for a drift if a unit root is not rejected in the first test, or the test for a deterministic time trend if a unit root is rejected in the first test.

b. The three numbers in each cell are, in order, the results when the nominal size of 10%, 5%, or 1% is used.

c. The size and power found on second test are calculated as the frequencies respectively of rejecting the true null hypothesis and of accepting the true alternative hypothesis for those situations in which stationarity status was chosen correctly.

With the true DGP in case (i) in the table there is a unit root with no drift, so the simulations are providing information on size on the first test (the unit root test) and size on the second test (the test for a drift term given there is a unit root). The results indicate the actual size matches the nominal size well on the first test, but the second test has actual size too high compared to the nominal level. The frequency of choosing the true DGP structure appears quite good, between 72% and 98%. With the true DGP in (ii) and (iii), there is a unit root with drift, so the simulations are providing information on size on the first test (the unit root test) and power on the second test (the test for a drift term given there is a unit root). Again, the size on the first test is what we expect for each of these true DGPs. The power is varying on the second test, but not in an unexpected way: when the drift term is strong as in case (ii), the power is 100%, and when it is weak as in case (iii), the power is notably weak. The frequency of choosing the correct model is strong when the power is strong as in case (ii) and is weakened by the weak power in case (iii).

With the true DGP in case (iv) there is stationarity around a nonzero constant, so the simulations are providing information on power on the first test (the unit root test) and size on the second test (the test for a deterministic trend with an otherwise stationary process). The power on the first test with these parameters seems good, but the most surprising aspect here is the actual size on the second test is zero for all three nominal size levels considered. Because of this, the frequency of choosing the true DGP structure is exactly equal to the power on the first test.

Case (v) is the same as case (iv) but with a very low rate of convergence for the stationary process (the coefficient on y_{t-1} is very low). Under these circumstances the true DGP is getting close to a random walk with drift, making the distinction difficult between the true DGP structure (stationary around a nonzero constant) and a DGP structure with drift-induced growth. However, since a DGP structure with drift-induced growth is not available as an option after the unit root has been rejected, the second test will mistake near drift-induced growth for time-trend induced growth more often in case (v) than in case (iv). This explains the high size values found in case (v) for the second test while in case (iv) they were all zero.

With the true DGP in case (vi) there is a trend stationary process, so the simulations are providing information on power on the first test (the unit root test) and power on the second test (the test for a deterministic trend with an otherwise stationary process). Here we see strong power in both cases due to the strong convergence parameter and the strong coefficient on the time variable. The frequency of choosing the correct model is high and equal to the power on the first test since the power on the second test is 100%.

Cases (vii), (viii), and (ix) are alterations of case (vi) on one parameter each. The changed parameter is made to be lower so we can examine situations where the true model is more likely not to be chosen. In case (vii) the convergence parameter is lowered in magnitude from -0.5 to -0.1, resulting in much lower power on the first test while the power on the second test remains at 100%. In case (viii) the coefficient on the time variable is reduce from 0.4 to 0.2 in comparison to case (vi). The power on the first test is not affected much by this change, but the power on the second test has gone down. Interestingly magnitude of the drop in the power on the second test varies from little with 10% nominal size to very much with 1% nominal size. This pattern is reflected in how the frequency of choosing the true DGP structure is reduced. Finally, in case (ix) the coefficient parameter for the time trend is reduced from 0.4 to 0.1 in comparison to case (vi). The pattern of changes observed between cases (vi) and (viii) are generally repeated between cases (vi) and (ix), although stronger in magnitude. In case (ix) there is very little power on the second test with 10% nominal size, and virtually zero power on that test with 5% or 1% nominal size.

III. Conclusions

One objective of this paper has been to evaluate a unit root model selection strategy suggested by Enders (2004) via Monte Carlo experiments. Our simulation results indicate serious mass significance problem if this strategy is used in conducting tests for unit roots. Our simulation results also indicate that utilizing prior restrictions to remove non-credible models, as suggested by Elder and Kennedy (2001), is exceptionally helpful, if not crucial, to have Dickey-Fuller unit root testing have empirical sizes close to their nominal counterparts. Since the Enders (2004) strategy does not utilize such prior restrictions, its use can be misleading as our simulations show.

We also investigate the size and power properties of the Elder and Kennedy (2001) unit root testing strategy when there is no knowledge of the growth status of the examined variable. Our simulations indicate that after the unit root status has been determined, the actual size of the subsequent test suggested by those authors (determining stationarity around a nonzero constant versus trend stationarity, or random walk versus random walk with drift) is rarely close to the nominal size, with there being the distinct possibility that the actual size will be extremely far from the nominal size when testing for a time trend after stationarity has been determined. The simulations also indicate that when trend stationarity is the true model, the test for inclusion of a time trend after the unit root has been rejected is more robust in its power to a low coefficient for the time variable when higher nominal size levels are used for both the unit root test and the trend test (e.g. there is more robustness at the 10% significance level than at the 1% significance level).

Acknowledgements

A version of this paper was presented at a conference entitled "The Cointegrated VAR Model: Methods and Applications", Copenhagen, 18-20 June 2006. The authors would like to thank the participants, especially David Hendry, Soren Johansen, and Katarina Juselius, for their useful comments and support. The usual disclaimer applies however.

References

Asteriou, D. and Hall, S. G. (2007) *Applied Econometrics: A Modern Approach*. Revised Edition. Palgrave MacMillan: New York.

Ayat, L. and Burridge, P. (2000): “Unit root tests in the presence of uncertainty about the nonstochastic trend”. *Journal of Econometrics* 95(1):71-96.

Dolado, J., Jenkinson, T. And Sosvilla-Rivero, S. (1990): “Cointegration and unit roots.”*Journal of Economic Surveys* 4(3): 249-73.

Dickey, D. A., and Fuller, W. A. (1979): “Distribution of the Estimators for Autoregressive Time Series with a Unit Root,*” Journal of the American Statistical Association*, 74, 427-431.

Dickey, D. A., and Fuller, W. A. (1981): “Likelihood Ratio Statistic for Autoregressive Time Series with a Unit Root” *Econometrica* 49, 1057-72.

Elder J. and Kennedy P. E. (2001) “Testing for Unit Roots: What Should Students Be Taught?” *Journal of Economic Education*, 32(2): 137-46

Enders, W (2004) *Applied Econometric Time Series*, Second Edition. John Wiley & Sons: United States.

Granger, C. (1981) “Some properties of time series data and their use in econometric model specification”*, Journal of Econometrics,*16, 121-130.

Granger, C. and Newbold, P. (1974) “Spurious Regressions in Econometrics”, *Journal of Econometrics*, 2, 111-20.

Harris, R. and Sollis, R. (2003) *Applied Time Series Modelling and Forecasting*. John Wiley & Sons, Chichester, West Sussex, England.

Holden, D. and Perman, R. (1994) “Unit roots and cointegration for the economist.” In B. B. Rao, ed. *Cointegration for the Applied Economist*. 47-112. New York: St. Martin’s.

Johansson, S. (1988) “Statistical Analysis of Cointegration Vectors”, *Journal of Economic Dynamics and Control*, 12, 231-254.

Johansson, S. and Juselius, K. (1990) “Maximum Likelihood Estimation and Inferences on Cointegration with Application to the Demand for Money”, *Oxford Bulletin of Economics and Statistics*, 52, 169-210.

MacKinnon, J.G. (1991): ”Critical Values for Cointegration Tests,*” Republished in; Long-run Economic Relationships, Readings in Cointegration,* Edited by Engle, R. F., and Granger, C. W. A., Oxford University Press.

Perron, P. (1988) “Trends and random walks in macroeconomic time series.” *Journal of Economic Dynamics and Control*, 12 (12): 297-332.

Phillips, P. (1986) "Understanding Spurious Regressions in Econometrics*." Journal of Econometrics*, 33, 311-40.

Phillips, P. (1987) Time Series Regression with a Unit Root, *Econometrica*, 55(2): 277-301.

Said, S. and Dickey, D. (1984) "Testing for Unit Roots in Autoregressive-Moving Average Models with Unknown Order." *Biometrica*, 71, 599-607.