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Abstract. *One of the claims of this paper is that three Austro-Swedish schools of economics provided much of the foundations for almost all of economic analysis developed after the second world war. Important representatives of the first school are Böhm-Bawerk and Wicksell, Schumpeter and Hayek of the second school, and Cassel, Wald and von Neumann of the third Austro-Swedish school of economics.*

However, there are serious omissions in all the three approaches to economic theory. The most striking is the lack of an analysis of the role of non-material and material public capital (or infrastructure) in the growth and development of economies. In this paper I demonstrate the theoretical approaches necessary for an extension of economic dynamics to develop the theories and models of these schools of economics.

In this paper I furthermore show that a proper refocusing on the time dimension can also shed light on the dynamics of economies in space. Three approaches are necessary for such a synthesis.

1. *Subdivision of products and systems of production according to their different and always positive durability, implying that everything produced is capital.*
2. *Subdivision of products according to the time used in their production.*
3. *Subdivision into private and public goods, allowing for non-linearity.*
4. *Allowing for differences in time scales of economic processes.*

With these distinctions it can be shown that the economic development in time ***and*** *space is determined by the impact of economies of scale, duration of the production process, durability of products and the - relative to most other kinds of capital - much slower growth of public capital (i.e. material and non-material infrastructure).*

JEL classification: O40; N01; L23; H41; F10; E58; E22; E10; D21; C62; B23; B13; R12; F12

Key words: Public capital; time scale, economic theory; Austro-Swedish schools

Time in economics

How is time to be represented in economic theory and models? The first and most obvious way is to represent time in accumulation of capital and other dynamic economic processes as a continuous variable. This implies that the processes are modeled as differential equations.

The second way is to represent the dynamic economic processes as a discrete set of periods (e.g. weeks, months, quarters or years), as illustrated by table 1 of this chapter.

The third way is to allow for many different continuous and interactive **time scales** of the dynamic economic processes.

The final important aspect of relevance in this context is to represent the dynamic characteristic of each product as the technologically and economically determined durability and the process of production in terms of duration of the process.

Building on Böhm-Bawerk´s (1889) concept of "roundaboutedness" in production Wicksell (1966) showed how the duration of production can be economically optimized subject to physical, biological or technological process constraints. But also the durability of the goods are determined by the duration and the resource accumulation of the production process.

In the sequel I will show how the durability of goods (seen a capital objects) will influence not only the dynamics of economies but also the spatial structure of production.

Time as the essential element of capital

The labour theory of value had been taken for granted by most of the classical economists. In Karl Marx´ Das Kapital it had been formulated as *the* fundamental proposition of economics having not only repercussions for most economic theorizing of those days but also for political interpretations of economics. Marx had assumed that capital was only an accumulation of labour during the period of its´ construction. Eugen Böhm von Bawerk (1891) set out to show the errors of the labour theory of value, when applied to the theory of capital. He introduced the idea of roundabout production and came to a provisional conclusion that the *time structure* of inputs is of essential importance in determining the value of capital..

Formulating a numerical example as in the table below he concludes that the decision of timing and the ruling rate of interest is essential to the optimal value of capital:

In an earlier chapter I called attention to the well-attested fact that the lengthening of the capitalist process always leads to extra returns, but that, beyond a certain point, these extra returns are of decreasing amount. Take again the case of fishing. If what we might call the one month's production process of making of a boat and net leads to the return of the day's labour being increased from 3 to 30,—i.e. by 27 fish,—it is scarcely likely that the lengthening of the process to two or three months will double or treble the return: Certainly the lengthening it to 100 months will not increase the surplus by a hundredfold. The surplus return—for there will always be a surplus return—will increase by a slower progression than the production period. We may, therefore, with approximate correctness represent the increasing productivity of extending production periods by the following typical scheme.

Without capital **With capital**/1 year	£15 35	**Increases** £20
2 years	45	10
3	53	8
4	58	5

5	62	4
6	65	3
7	67	2
8	68:10s.	1:10s.
9	69:10s.	1
10	70	0:10s.

His conclusion is: *The rate of interest under the foregoing assumptions is limited and determined by the productivity of the last prolongation of the production period which is still economically permissible and that of the next prolongation which is not so permissible. (Böhm-Bawerk, Positive Theory of Capital,1891, book VIII, section VII.I.9.*

The Swedish young economist Knut Wicksell was deeply influenced by Böhm Bawerks analysis of capital and time**:** *I remember as if it had happened yesterday, a day 25 years ago, when I in a bookstore window in Berlin –where I was living on a Gustaf Lorén scholarship – for the first time read the book title Positive Theorie des Kapitales by Eugen Böhm v. Bawerk. ... this work was to me a revelation.* (Wicksell, 1914, p.322 Translated from Swedish)

Wicksell, who was a mathematician, understood that Böhm-Bawerk´s tables could be generalized into a mathematical optimization problem. This became the famous wine maturation problem, in which he set out to determine the economically optimal duration of storing a wine. He then assumed that the value of the wine would be continuously growing if the wine was stored. During the process of storage there is of course a biological process during which solar energy and the activity of yeast and other components of the wine contribute to the growing value, which is finally determined by the utility to the consumer as expressed by the willingness to pay for the matured wine. The constraining factor was the opportunity cost of storage, determined by cost of labor, human capital of the wine producer, the price of the storage property and the market rate of interest.

It is assumed that the owner just wants to harvest once and V(T) is the value of the wine if it is brought to the market at harvesting time T.

The present value (PV) of stopping the maturing at time T is determined as:

Maximize PV, where $PV = (p-c)*V(T)*e^{-rT}$

Necessary condition of optimality of harvesting time is V´/V=r

The necessary condition of optimal economic duration of storage thus says that the storing should be stopped when:

The rate of growth of value equals the rate of interest.

A number of analysts have claimed that this is a special dynamic case, only relevant for point input, point output decision problems. However, this is wrong. It is shown that the condition holds also for continuous sequences of harvesting of some growing biological resource over time (e.g. trees in a natural forest, or fish in the sea).

Maximize harvesting income = ∫pux(exp(-rt))dt

Subject to the growth condition:

$dx/dt=ax - bx^2 - ux$

u= the rate of harvesting

x=the stock of the biological resource

Assumptions: infinite time horizon and constant price

Maximizing the Hamiltonian

$H = pux(\exp(-rt)) - \lambda(ax - bx^2 - ux)$

leads to an optimal rate of harvesting at each instance of time.

Optimality requires that $\lambda'/\lambda = r$

This again means that the rate of growth of value should equal the rate of interest at the optimal rate of harvesting.

The debate about the Marxian labor theory of value should by then had come to an end. Not only would land (natural resources) be an indispensable fundamental factor of production beside labor as had already been shown by Johann Heinrich von Thünen (1826). Capital could not be reduced to labor by any procedure. Time had been shown to be the crucial variable in the determination of the optimal value and rate of capital accumulation.

Durability, capital, production and economic growth

The average durability of products (i.e. capital entities) of the macro economy can be determined by observing that the value of capital is a stock concept, observable at any instant in continuous time. The value of production is a flow concept, requiring measurement over some time period. The value of the stock divided by the value of the flow thus has the dimension of time (Hawkins 1948, Hawkins and Simon, 1949).

This implies that the average durability of the goods (i.e. capital) is determined by the ratio of the value of capital to the value of production. This, in its´ turn, implies that the average durability of capital is determined by quantities of capital and production as well as their prices. Durability of capital is closely related to depreciation. If the durability is known with certainty to be T years, then the optimal depreciation per year is 1/T. If the durability is uncertain but has a known mean value, then the depreciation can be determined with the aid of entropy theory and be in this case, the depreciation rate is a constant fraction of the net asset value (Lev B.,Theil H. 1978).

Assume that total production in a one-sector economy is subdivided into currently used production and production for future use (i.e. investment). Current production requires inputs at the rate ay, where a gives the necessary input per unit of output y. Investments are determined by bgy, where g signifies the rate of growth of y and b is capital requirements per unit of increase of production. As b divided by a equals the durability of the product, $b=\tau a$, where τ is the durability of the product (y). Thus:

$$y = ay + g\tau ay$$

Furthermore, it is a long run equilibrium condition that the price equals cost of current inputs as well as capital cost:

$$p = pa + rp\tau a \quad \text{or} \quad (1-a)/\tau a = r$$

This implies that $g = r$ or

The rate of interest must equal the rate of growth in an economically sustainable equilibrium of this simplified economy.

However, it can be shown that this result is also valid for an economy with any number of sectors and a technology represented by the input-output and capital–output matrices A and B where B= TA with T being a diagonal matrix of durabilities of products:

$x = Ax + gBx$; The primal condition of a general growth equilibrium

$p = pA + rpB$; The dual condition of a general growth equilibrium where x = production vector

p = price vector

A = the n*n semi-positive input/output matrix

B = the n*n semi-positive capital/output matrix = TA.

g = the maximal rate of growth at the general growth equilibrium

r = the minimal rate of interest at the general growth equilibrium

The longer the durability of any one product, ceteris paribus, the lower would the rate of interest and growth would have to be. The only way of extending a durability without a decline in the rate of interest and growth would be a sufficiently large reduction of the use of current inputs.

Cassel, Wald, von Neumann and the birth of general equilibrium theory in Vienna

Although Carl Menger´s, Böhm Bawerk´s, Wicksell´s and Schumpeter´s books and articles played a great role in the formation of economic theory in Austria and Germany, the most widely used *textbook* in economics in the German speaking world from 1917 onwards was Gustav Cassel´s Theoretische Sozialökonomie (1917). Karl Menger, the son of the economist Carl Menger, assembled some of the great mathematicians in his Mathematische Colloquium in Vienna in the early 1930s. This was no haphazard event. Menger's efforts were oriented to fruition of his father Carl Menger´s – Böhm-Bawerk´s – Wicksell´s and Cassel´s theories into a consistent mathematical economic theory. His focus was on axiomatization of the fundamental ideas of Austrian economics. His Kolloquium became the birth place for a formalization of static and dynamic general economic equilibrium as laid out by Cassel in his textbook. Cassel, of course, had accepted the idea that utility is essential in determining economic decisions by the individual consumer. But he was against the idea that there is an absolute need for a deduction of aggregate demand from the theory of individual consumer utility. It was, according to Cassel, sufficient to have knowledge of all demand and supply functions (dependent on all prices) in order to generate a general static equilibrium.

Wald (1935) proved that this was indeed the case.

If aggregation of individual utility or demand functions could be shown to be possible this would also cover the general equilibrium problem as formulated by Menger (1871) and Walras (1874).

John von Neumann (1936) went along the route opened by Wald to show that a dynamic model based on the same approach combined with the growth model formulated in Cassel´s textbook. That growth model saw the equilibrium rate of growth as determined by the willingness to abstain from current use of resources by saving and the technologically determined capital-output ratio. This is in its´ essence the same growth model as formulated above. Von Neumann proceeded to generalize this model into a theory of an economically sustainable dynamic general equilibrium, based on the use of the saddle point theorem that he had proved in the 1920s.

He introduced time into his theory in two ways. First, he formulated the basic model as discrete period dynamics. Second, the durability of all products were handled as rates of depreciation between periods.

Von Neumann assumed joint production in order to treat depreciation efficiently in his model

THE VON NEUMANN MODEL OF CAPITAL AND GROWTH

q = vector of outputs
p = vector of prices
α = 1+rate of growth
β = 1 + rate of interest
A = mn matrix of inputs
B= mn matrix of outputs
The model allows for joint production which means that the amount of a capital product shrinks as a consequence of depreciation

The equilibrium is a saddle-point solution determining the equilibrium price and quantity vectors . At the same point the minimum rate of interest and the maximum sustainable rate of growth are equalized.

$$\mathbf{q}^{\mathrm{T}}\mathbf{B} \geqq \alpha\mathbf{q}^{\mathrm{T}}\mathbf{A}. \quad (1)$$

$$\mathbf{B}\mathbf{p} \leqq \beta\mathbf{A}\mathbf{p}. \quad (2)$$

$$\mathbf{q}^{\mathrm{T}}(\mathbf{B} - \alpha\mathbf{A})\mathbf{p} = 0. \quad (3)$$

$$\mathbf{q}^{\mathrm{T}}(\mathbf{B} - \beta\mathbf{A})\mathbf{p} = 0. \quad (4)$$

$$\mathbf{q} \geq \mathbf{0} \text{ and } \mathbf{p} \geq \mathbf{0}. \quad (5)$$

An example: in the process of making paper, wood, electricity and machines are used as inputs at the start of the process.

At the end of the process a **joint product** consisting of paper, and machines, older and thus smaller in capacity is the vector of outputs.

The sustainable equilibrium is a saddle-point solution determining the equilibrium price and quantity vector . At the same point the minimum rate of interest and the maximum sustainable rate of growth are equalized. This is consistent with Wicksell´s and Böhm Bawerk´s results for micro economic growth processes. The weakness from a neoclassical point of view of von Neumann´s theory is the lacking foundation of utility of the decision makers of the model, which might go back to the views of Gustav Cassel, who had dismissed the importance of individual utility functions

The work of the Kolloquium in Vienna came to an end in 1936. The participants had by then created a foundation for the development of modern mathematical economics with its´ reliance on the use of saddle points and fixed point theorems. This includes the use of these ideas in game theory, as created by John von Neumann and Oscar Morgenstern (1944) and in general static equilibrium theory, assuming utility maximizing consumers, as reformulated by Debreu (1959). It is furthermore reasonable to regard von Neumanns introduction of inequalities in saddle point theory as one of the main preconditions for linear and non-linear programming theory and modeling. The other mathematical set of theorems to be used in programming theory was the book Inequalities by Hardy, Littlewood and Polya (1933).

Determination of the rate of interest

Böhm-Bawerk gave three reasons why the interest rate has to be positive. First, people’s marginal utility of future income is lower than the same present income if they expect economic growth. Second, psychologically most people are impatient and simply prefer a given income now instead of waiting for it. These two reasons imply a willingness to pay a positive interest rate to get access to loans in the present and a corresponding requirement to be paid an interest rate if they give a credit to someone. The third reason is according to Böhm-Bawerk the technological advantages of a roundabout production process as described above.

Wicksell essentially accepted these micro-economic reasons but insisted on seeing the interest rate as a macro-economic variable, exogenous to the individual households and firms. According to Wicksell one must distinguish between the *natural (or normal*) rate of interest and the rate of interest, determined by supply and demand for credit in the money market. A steady state macro-economic equilibrium with a constant level of prices requires the money market rate of interest to be equal to the natural rate of interest. A proper monetary policy by the central bank would then mean a policy of adjustment of the central bank rate of interest to the rate of inflation.

If the general price level would start increasing, the proper response of the central bank would be to increase the rate of interest and to decrease it in a deflationary situation Wicksell(1906,1966). Wicksell´s analysis was essentially dynamic: A monetary rate of interest lowered below the natural rate of interest would trigger an increase of investments. This would mean an increase of total demand above the capacity to supply. The response in the market would then initially be an increase of prices of machinery, building material and other investment goods as well as the wages of construction labor. The increase of laborer´s income would imply increasing demand for consumer goods and a corresponding decrease of real savings. The prices of consumer goods and the wages of other workers would thus also have to be increased and this cumulative inflationary process would be going on as long as the monetary rate of interest would be kept lower than the natural rate of interest. The only way of braking this inflation process is by increasing the monetary rate of interest to the level of natural rate of interest at which the willingness to save would be in balance with the marginal productivity of capital.

Recent decades have shown a return from Keynesian and naïve monetary economic policies to Wicksellian central bank strategies in many countries.

Hayek accepted Wicksell's analysis of the relation between the natural and the monetary rate of interest, but tried to extend its importance considerably..

Hayek agreed with Wicksell in seeing a too low rate of interest set by the banking system as a reason for a cumulatively rising price level. But Hayek went much further than Wicksell, making the monetary policies the central mechanism to be used to combat inflation as well as unemployment, i.e. the central mechanism regulating the business cycle.

Wicksell thought of his contribution as a theory of the general price level and of how to control it. Monetary effects on relative prices, as assumed by Hayek, on resource allocation and the business cycle fluctuations would to him be secondary only. Wicksell disapproved of the use of his theory by Mises and Hayek, who had adapted his basic analytical idea to business cycle theory.

Schumpeter, who in many respects admired Böhm-Bawerk and Wicksell, had a dramatically different view of the rate of interest. In the static and no growth equilibrium of an economy the interest rate would automatically fall to zero, because he saw a positive rate of interest as relevant only to a developing economy. (Schumpeter, 1912, Haberler 1951). This view is consistent with the equilibrium theory of economic growth, as formulated by von Neumann. But a closer scrutiny of that model also reveals why duality requires the interest rate to be equal to the growth rate on the equilibrium trajectory. The model is based on two important and simplified assumptions. First, there is no uncertainty in the economic system modelled. Second, households are treated as any other sector of production. There is no room for trade-off between future and current consumption and thus no consideration of the two first reasons for a positive rate of interest.

If decision makers are uncertain about the future, for example as a consequence of variations in the weather, health of workers and other natural conditions influencing production or consumption, there would be a need for a positive rate of interest in order to bridge the gap between lenders and borrowers. We can thus conclude that a general economic equilibrium with uncertainty requires that *the rate of interest is kept above the rate of economic growth*.

Capital controversies

Böhm-Bawerk (1891) as well as Hayek (1940) had been struggling with the creation of a complete theory of capital, including a disaggregation of all capital goods and a consistent method of aggregation into a macroeconomic consistent capital stock. The essence of their problem was the heterogeneity of the capital goods. This was later followed up by the Marxist influenced capital controversy of the Cambridge/Italy school (Pasinetti,1969).

I am convinced that the problem of capital aggregation can be resolved, as soon as we accept the necessary uncertainty or risk of capital investments and the determination of capital prices in the markets for financial capital, especially the stock market.

It is reasonable to assume that macroeconomic capital equals value of capital at prices as expected by financial investors. A firm, traded on the stock market, is, as discussed below, essentially an aggregated value of different capital goods, including knowledge capital in different forms. Modern theory of financial markets claims that the total equilibrium value of capital of any traded firm is determined (as an average over some period of observation) in the markets for securities and bonds, taking expected prices, perceived risk and real rate of interest into consideration.

The theory of financial markets was initially formulated by Markowitz (1952) and further developed by Modigliani and Miller (M-M) (1958), Sharpe (1964), Lintner (1965), and Mossin (1966).

Their claim is that the capital market is M-M-efficient implying that the totality of all capital allocation opportunities can be captured by the expected return r(m)) and the risk or standard deviation (β(m)) for the market portfolio of all traded instruments. The value of a firm (seen as an aggregate of material and non-material capital) can be determined in an analogous way.

The risk-free or deterministic capital allocation would give $\beta(0)=0$ and $r(0) \geq g$.

From this follows the conclusion that the heterogeneous capital value, aggregated by the firm and valued in the stock market, when divided by the scale of production of the firm would generate the average durability of the capital, invested in the firm.

The firm as an emergent order of capital

The theory of the firm as formulated by Oliver Williamson (1981) and others, being based on Ronald Coase´s transaction cost assumptions(1937), is a useful starting point for an analysis of the formation of firms, but a starting point only. It says nothing about the best organization of material and human capital in the organization of the firm.

A haphazard arrangement of the carriers of human capital and the machinery and other material capital will not give the same high level of output as an arrangement generating a high level of profits to the entrepreneur. However, it can be shown that maximizing the profitability of interactions between a large number of such discrete objects can often not be found, even with the aid of powerful computers.[2] For a firm with only 10 groups of employees to be allocated to 10 different tasks there are in fact 3.6 million possible assignment patterns and the number of possible patterns increases with the factorial of the number of tasks and employee groups. Already with a size of 20 tasks and employee groups the total number has risen to 2 432 902 008 176 640 000 of possible patterns of assignment. With advantages of interactions there are usually a large number of local profit maxima in this class of problems and the search for the global maximum is thus very hard.

The formal non-linear optimal assignment problem can be approached as in the following integer programming model, proposed by Andersson and Kallio (

Maximize $x' S x + Rx$

Subject to $\sum(j)x_{ij}\leq 1$; Specialists available

$\sum(i)x_{ij}\geq 1$; Tasks to be fulfilled

x =(0 or 1)

S is a non-definite matrix giving the positive or negative advantages of collaborating between each pair of employees and R gives the revenue effects of each individual if operating a task on her own.

We developed an computer algorithm that would search for a local optimum when started from randomly selected starting points. The numerical procedures found different local optima, quite different from each other and for large problems the number of such local optima could be extremely large. There is no assurance that a global optimum would be found in finite computer time with a reasonably large number of feasible assignments.

However, the probability of finding solutions close to the global maximum is vastly increased if different firms in the same market for goods are experimenting in different ways with their organization of production. Competition will then in the long run

reveal the best practice firms with a superior organization in terms of profitability.

An evolutionary procedure called the *Patch Procedure,* where a patch can be a predetermined team of employees, has been developed by Stuart Kaufmann and his associates (1996):

The results hint at something deep and simple about why flatter, decentralized organizations may function well: contrary to intuition, breaking an organization into "patches" where each patch attempts to optimize for its own selfish benefit, even if that is harmful to the whole, can lead, as if by an invisible hand , to the welfare of the whole organization. The trick, as we shall see, lies in how the patches are chosen. We will find an ordered regime where poor compromises for the entire organization are found, a chaotic regime where no solution is ever agreed on, and a phase transition between order and chaos where excellent solutions *are found rapidly*(Kauffmann, p. 147)….He concludes: *Therefore, as a general summary, it appears that the invisible hand finds the best solution if the coevolving system of patches is in the ordered regime rather near to the transition to chaos*.(Kauffman, p.264)

At the end of such an evolutionary process the superior firms with their structure of teams will have a capital value far above what would be indicated by their book-value of purchased machines and human capital.

The part of the capital value that cannot be easily accounted for is often in accounting practice called Good Will Value:

Business goodwill is a key intangible asset that represents the portion of the business value that cannot be attributed to other business assets.

Put differently, business goodwill reflects the synergy among the various assets used by the business to produce income: in a well-run business the whole is greater than the sum of the parts. (http://www.valuadder.com: ValuAdder Business Valuation Tools, Haleo Corporation, 2012,)

The only way tangible and intangible, organizationalcapital is properly valued is by the valuation of the firm in the stock market or at the instance of sale of the firm as an organized entity.

Footnote 2: If we assume indivisible units of machines and humans and that the productivity of a machine or a human (x(i)) depends on interaction with (x(j)) and if these interaction net benefits can be captured by the quadratic form x´Cx, then there is no simple incentive mechanism or computerized search algorithm that would provide the route to a global maximum for most interaction matrices C.(Koopmans T. and Beckmann M. 1957)

Durability of products and patterns of location of production

The problem of the spatial structure of production is related to the sustainable scale of each firm and total demand for their products. The sustainable scale of a firm is determined by the minimum of long term total average cost, including interest on capital, transaction and transport costs. A long term equilibrium of the firm requires the price to be charged to correspond to this minimal long run average cost.

In order to determine the impact of product durability on the spatial structure of production we need to specify the long run average production cost function (APC) and its´ dependence on the scale of production, x. A simple but mostly realistic assumption is to subdivide the total production cost into fixed cost (F) and variable cost (V(x)). Fixed cost is the cost of all capital of the production unit and is independent of the scale of operation as soon as the production unit has been established. For simplicity we assume that the production unit is identical to the firm.

This analysis provides a convenient connection with Austrian economics and specifically with Böhm-Bawerk and with Wicksell. Their analysis of the importance of roundabout production and duration of the production process increases the amount of capital needed and thus of the fixed cost of production. This is especially pronounced in knowledge intensive production, needing a long period of research and development before actual production can occur. Typical examples are the pharmaceutical and advanced electronic industries, which regularly invest more than twenty per cent of their sales value in creation and innovation of new products and associated production equipment.

The variable cost is normally monotonously increasing with the scale of operation up to the capacity limit of the capital of the firm. We consequently assume that the optimal scale of operation is smaller than or equal to that upper limit. The simplest variable cost is the linear case $V(x) = vx$. The total production cost function would thus be $TPC = F + vx$; and the average production cost function would thus be $APC = F/x + v$.

Transport and transactions cost are caused by the contacts between the firm and its´ customers. If customers are more or less evenly spread around the firm the total cost of such contacts would increase progressively with the increase in the scale of production and sales. This means that the average cost of transactions and transport would be increasing with the scale of operations. A simple, yet reasonable approximation is a an average transport and transaction cost function ATC = kx. The term k can be decomposed into cost per unit of shipments ,a, and the frequency of contacts, which is inversely depending on the durability, t, of the product. The average transport and transaction cost, is thus ATC = (a/t) x. The longer the durability the lower will be the average transport and transaction cost.

The total average cost A equals the sum of average production cost APC and average transport and transactions cost ATC:

A= F/x + c + (a/t) x. The minimum average cost requires that the derivative of A with respect to the scale of production is set equal to zero. This implies that the optimal scale of production is:

$$x(opt) = \sqrt{\frac{Ft}{a}}\,.$$

The optimal scale of production of the firm is thus increasing with increasing fixed cost of production (primarily of capital and land in the roundabout production process) and of increased durability of the product.

It ought to be stressed that the fixed cost is an increasing function of the duration or roundabout degree of production.

The optimal number of firms is determined by the total scale of the market. The maximal total market scale is today the world market as it is integrated by information and transport networks. The existence of an accessible world market is a precondition for perfect – or at least free - competition for most tradable goods.

The total number of firms in the world market, N, for a good is then determined as:

N = Total demand / x(opt), which implies that the total number of firms is decreasing with the fixed cost of the representative firm and also decreasing with the durability of the good being analyzed.

Fixed cost, as influenced by the duration of the production process, and durability of the good produced reinforce each other in decreasing the number of firms if the demand of the world market is given. In some cases the number of firms is so severely constrained, that the assumption of perfect or free competition cannot be upheld even if the product is globally traded. Examples are trains, ships, airplanes, nuclear reactors, which are produced only in a few locations, serving a global market.

The number of firms is thus determined by this procedure, but not the locations in geographical space. For that a connection with the theory of location and trade is needed. The theory of location and trade summarizing the contributions by Ricardo, von Thünen, Heckscher and Ohlin and Beckmann is the recent variational inequality model as formulated by Anna Nagurney (1999). In this model demand at each location and supply in each location are represented by the respected prices announced in the locations. An increased flow of a good from a location to another requires the price difference to be larger than the sum of transport and transaction costs associated with a unit trade flow between the two locations.

The pattern of location and trade flows comes to an equilibrium when each good price difference is equal to (or smaller than) the sum of transaction and transport costs. As we have seen above the durability of each good determines their transaction and transport cost. The larger the durability of the good the smaller is this cost. Trade will increase until there are no price differences between different locations for the limiting case of extremely large durability of a good. For goods of extreme durability and low cost of transportation and transactions a law of one price is ruling.

Infrastructure – capital that is durable and public

The static and dynamic equilibrium theories and models developed by the different Austrian schools have provided the fundaments for modern economics. But they are inadequate in one important respect. They cannot handle the dynamics of durable public goods or infrastructure, for example constitutions, scientific knowledge or communication networks. In the Austrian theories infrastructure is an exogenously determined stable stage on which the economic games are played. The economic and social consequences of different infrastructural stages are discussed, for instance in the analysis of socialism versus the market economy in the writings from Böhm-Bawerk and Wicksell to Mises and Hayek. But a dynamic analysis of the interdependent evolution of the of infrastructure and economic games is essentially lacking, even if traces of such an analysis can be found in Schumpeter´s Capitalism, Socialism, and Democracy (1942,1950).

The reason for this omission is quite clear. The necessary mathematical foundation for such an interdependency analysis did not exist before the 1960s. The first attempt to analyze catalytic and other collective phenomena was by the Field Medalist Reneé Thom in his *Structural stability and morphogenesis : an outline of a general theory of models (1989).* In this book, originally published in 1972, he showed how collective phenomena could be modelled with singularity theory and applied to biological phenomena, such as the simultaneous blooming of a certain species by the influence of the slowly rising temperature.

In the 1950s and 60s physicists developed the theory of the laser. The German physicist Hermann Haken was one of the central analysts developing this theory. A central part of this work wasd what later came to be known as *Synergetics*. This theory was later to be published in the 1970s (Haken ,1977). Haken showed that predictability can often be achieved by subdividing dynamic processes into widely separated time scales. A general equilibrium of the combined dynamic system is a possibility, if the slowly changing variables are causally impacting a large number of rapidly changing variables.

The institutional and material infrastructure is by definition such a collective (or public) variable moving on a qualitatively slower time scale than the private goods allocated on the markets:

- Infrastructure is simultaneously used by many firms or households
- Very durable, compared with other capital.

The following dynamic model of a market economy illustrates the power of subdividing the economic system into widely different time scales .

The dynamics of the markets for ordinary goods is determined by differential equations determining the price trajectories of all goods:

$dp/dt = f(p,k,A^*)$;

$s(k)\, dk/dt = g(p,k,A^*)$;

where

p = a vector of prices of ordinary market goods (including factor services),

k = a vector of capital or investment goods.

A*= infrastructure as represented by accessibility of knowledge over networks

s(k) = a parameter. Smaller than 1, representing the slow speed of capital growth

The development of infrastructure (as represented by accessibility to fundamental knowledge) can be represented by the equation:

$s(A)dA/dt = m(p,k,A)$;

where s(A) represents the very large durability of infrastructure, indicating that s(A) is a very small, positive number, possibly in the order of 0.01 or lower.

This implies that in the time frame of the other variables of this system dA/dt can be set approximately equal to zero, **most of the time**, but not always. Very rarely. the fast and slow processes will be synchronized and the whole system will go into a period of creative destruction, eventually to come into rest at a new economic structure.

We thus have a dynamic system:

$dp/dt = f(p,k,A^*)$,

$s(k)\, dk/dt = g(p,k,A^*)$;

to be solved subject to the **temporary constraint**:

$m(p,k,A^*) = 0$.

For systems of this kind we can apply Tikhonov´s theorem (Sugakov, 1998):

Assume a dynamic system of N ordinary differential equations, which can be divided into two groups of equations. The first group consists of m fast equations, the second group consists of m+1, …,N slow equations.

Tikhonov´s theorem states that the system:

dx(i)/dt = f(i)(x,g); i=1,…,m, (fast equations)

f(j)(x,g)=0; j=m+1,…,N, (slow equations),

has a solution under certain economically reasonable conditions.

For each position of the slow subsystem, representing the dynamics of infrastructure, the fast market subsystems have plenty of time to stabilize. Such an approximation is called adiabatic (Sugakov,1998).

In the very long run dA/dt cannot be assumed to be zero and the system as a whole would then cease to be as well behaved as in the short and medium terms of dynamics.

The system would in the very long term have all the bifurcation properties , typical of non-linear, interactive dynamic systems. Between periods of change of the economic structure, there would be periods of stable markets growth equilibrium.

Most neoclassical economists have become skeptical about the possibility to mathematically model the dynamics of economic system. Modern mathematical theory of dynamic systems supports this view. General equilibrium theory, as conventionally formulated by Arrow, Debreu and others, are in fact not general enough to be expandable into dynamic systems (or combined spatial and dynamic systems).

However, I have shown above that this impossibility can be resolved if the dynamic models of the economy have proper distinctions between the time scales of markets for services, non-durable goods, capital accumulation and the slow changes of the infrastructural stage on which the markets operate.

Conclusions

The theories and observations of the role of the time dimensions of the early Austrians, including the Swedish economist Knut Wicksell, are fundamental to our understanding of the dynamic economic processes. The economic importance of a proper choice of duration of a roundabout process was proposed by Böhm-Bawerk and Wicksell. With the increasing importance of scientific and industrial research and technological development this issue had become increasingly important.

But duration of the production process must be complemented by the durability of the goods produced. Durability of the goods – and all goods are also to different degrees capital – is a determinant of many aspects of the economy as a growing system.

The durability of goods contributes in determining:

- the macro-economic capital-output ratio,
- the maximal rate of growth of the economy,
- the minimal rate of interest,
- the optimal number of firms, and thus
- the spatial extent of the competitive market and
- the pattern of location and trade.

The extreme durability of institutions, networks and other infrastructure is especially important as its provides a theory of modeling of complex dynamic systems. We thus have the methodology needed to avoid the limitations of standard general equilibrium theory in favor of a more complete and yet formal dynamic general theory of the evolving market economy.

References.

Andersson, Å.E. (2011) Creative people need creative cities. *Handbook of Creative Cities* (Andersson, D.E., Andersson, Å.E. and Mellander, C. Eds.) Cheltenham, UK: Edward Elgar

Andersson Å.E. and M. Kallio (1982) A Mathematical Programming Approach to Land Allocation in Regional Planning, IIASA, WP, Laxenburg, Austria

Böhm-Bawerk, E. (1959-1921a). *History and Critique of Interest Theories*. Libertarian Press, South Holland, Illinois. (First published as *Kapital und Kapitalzins*.) Volume 1: *History and Critique of Interest Theories*, 1884. Volume 2: *Positive Theory of Capital*, 1889. Volume 3: *Further Essays on Capital and Interest* was first published as appendixes to Volume 2 of the 1909–1912 edition, and was printed in a separate volume in 1921b

Burmeister, E. (1974). Synthetizing the Neo-Austrian and Alternative Approaches to Capital Theory: a Survey. *Journal of Economic Literature*, pages 413-56

Coase, R.(1937), The Nature of the Firm, *Economica* **4** (16): pages 386–405,.

Debreu G. (1959). *Theory of value* An Axiomatic Analysis of Economic Equilibrium. New Haven and London, Yale University Press

Dorfman, R. (1959b). Waiting and the Period of Production. *Quarterly Journal of Economics*, pages 351-72.

von Haberler,G. (1937) *Prosperity and Depression: A Theoretical Analysis of Cyclical Movements* (Geneva: League of Nations, 1937

Haken, H. (1977), *Synergetics, An introduction. nonequilibrium phase-transitions and self-organization in physics, chemistry and biology.* Berlin:Springer

Hawkins D. (1948), " Some conditions of macroeconomic stability," *Econometrica* October, pp. 309-322

Hawkins D. and HA Simon,(1949) "Note: Some conditions of macroeconomic stability," *Econometrica 17): 245-248.*

Hayek, F. A. (1931). *Prices and Production*. London:Routledge & Kegan Paul.

Hayek, F. A. (1941). *The Pure Theory of Capital*. London: Routledge and Kegan Paul.

Hayek, Friedrich A. 1945. “The Use of Knowledge in Society.” *American Economic*

Review (September): 519-30

Hicks, J. (1970). A Neo-Austrian Growth Theory. *The Economic Journal*, pages 257-81.

Hicks, J. (1973). *Capital and Time*. Clarendon Press, Oxford.

Janik, A. and Stephen Toulmin. (1973) *Wittgenstein's Vienna*. Ivan R. Dee Publishers, Chicago, Ill.

Jevons, W. S. (1871-1970). *The Theory of Political Economy*. Penguin Classics. London

Kauffman, S. (1995). At Home in the Universe. The Search for Laws of Self-Organization and Complexity. Penguin Books. London

Knight, F. (1934). Capital, Time, and the Interest Rate. *Economica*, pages 257-86.

Lev B., Theil H.(1978). A Maximum Entropy Approach to the Choice of Asset Depreciation. *Journal of Accounting Research,* Vol. 16 No. 2, pages 286-93.

Lintner, J. (1965). The Valuation of Risk Assets andthe Selection of Risky Investments in Stock Portfolios and Capital Budgets. *Review of Economics and Statistics, 73*, 13-37.

Markowitz, H. (1952). Portfolio Selection. *Journal of Finance, 12,* 71-91.

Marschak, J. (1934). A Note on the Period of Production. *The Economic Journal,* pages 146-51.

Marshall, A. (1890). *Principles of Economics*. Macmillan, London, 1 edition. 2nd. ed. 1891, 4th. ed. 1898, 8th. ed. 1920.

Menger, C. (1871, 1981). *Principles of Economics*. New York: New York University Press. English Translation of Grundsätze der Volkswirtschaftslehre, by James Dingwall and Bert Hoselitz. .

von Mises, L. (1912*). Theorie des Geldes und der Umlaufsmittel.* Leipzig: Duncker und Humblot in English: von Mises L. (1953*), The Theory of Money and Credit,* New Haven,: Yale University Press Edition Haven

Modigliani, F. & Miller, M. (1958). The Cost of Capital, Corporation Finance, and the Theory of Investment. *American Economic Review, 48* (3), 261-297.

Mossin, J. (1966). Equilibrium in a Capital AssetMarket. *Econometrica, 34* (4), 768-83

von Neumann, J. and O. Morgenstern. *1944. Theory of games and economic behavior.* Princeton, N.J.: Princeton University Press

Morgenstern, O. (1935). Zur Theorie der Produktionsperiode. *Zeitschrift für Nationalökonomie*, (Band VI):196-208.

Nagurney, A. (1999). *Network Economics: A Variational Inequality Approach*. Amsterdam, Kluwer Academic Publishers.

Pasinetti, L. (1969), "Switches of Techniques and the “Rate of Return” in Capital Theory", in *The Economic Journal*, vol.79, pp. 508–531

Samuelson, P. A. (2001). A Modern Post-Mortem on Böhm’s Capital Theory: Its´ Vital Normative Flaw Shared by Pre-Sraffian Mainstream capital-theory. *Journal of the History of Economic Thought*, 23(3):301-17

Sharpe, W.F. (1964). Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. *Journal of Finance, 19*(3), 425-42.

Carl Schorske, C. (1981) *Fin de Siècle Vienna*

Schumpeter, Joseph A. 1912. Theorie der wirtschaftlichen Entwicklung. Leipzig: Duncker und Humblot

Schumpeter, Joseph A. 1942, 1950. *Capitalism, Socialism, and Democracy*. New York: Harper & Row

Thom, R. (1989) *Structural stability and morphogenesis : an outline of a general theory of models*, Reading, Mass. : Addison-Wesley Pub.

Wald, A.,(1936,1951), On Some Systems of Equations of Mathematical Economics", *Zeitschrift für Nationalökonomie*. (translated in *Econometrica, Vol.19 (4), p.368-403,1951*.)

Walras, L. (1874, 1954, Engl. transl.)(Éléments d'économie politique pure, ou théorie de la richesse sociale) Elements of Pure Economics, or the theory of social wealth, transl. W. Jaffé, London: Allen and Unwin

Wicksell. K. (1966), *Föreläsningar i nationalekonomi.*
Lund:Gleerups, Femte upplagan.

Wicksell, K. (1967). *Lectures on Political Economy*. Reprints of Economic Classics. Augustus M. Kelley Publishers.

Wicksell Knut, (1914), Lexis och Böhm-Bawerk, *Ekonomisk Tidskrift*
Årg. 16, häfte 11 pp. 322-334
Williamson Oliver E., (1981), "The Economics of Organization: The Transaction Cost Approach". *The American Journal of Sociology* **87,** pp. 548-577