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The income return to entrepreneurship – theoretical model and outcomes for Swedish regions

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Abstract: This paper investigates the income return to entrepreneurship and wage employment by means of Lazear's model of occupational choice. The income return of an actor is defined as the income resulting from the preferred occupational choice divided by the hypothetical income he or she would earn, if for some reason forced to make the opposite choice. We analyze this by deriving theoretical implications of assuming that the skill strengths in Lazear's model are Fréchet distributed. In the empirical part of the paper, data from the Swedish employment register for individuals having a university exam in electrical engineering is used to compute the return to self-employment and wage employment. The results are reported by a division of Sweden into three regions and for different subgroups by sex, age and earlier experience of self-employment. Our computations show that the average return to entrepreneurship is less than 5 percent for most subgroups of self-employed and the average return to wage employment over 50 percent for more than half of the subgroups. The return to the self-employed hiring at least one person is higher in the Stockholm than the other regions. Self-employed have higher returns if they are males, 45 years or older, or if they have earlier experience of self-employment. The ratio between average income of self-employed and wage employed is less than 1 for almost all subgroups in all regions consistent with our theoretical model. Together with the similarities between computed and observed income distributions for self-employed and wage employed, this supports the Fréchet assumption in our application of Lazear's model.

Keywords: entrepreneurship, self-employment, wage employment, regional occupational choice, income return, skill distribution, Fréchet distribution

JEL codes: M13; J24; J3; L26

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The income return to entrepreneurship – theoretical model and outcomes for Swedish regions

1. Introduction

This paper investigates the income return to entrepreneurship by means of Lazear's model for occupational choice, Lazear (2004, 2005). His model states that an individual will choose to become entrepreneur whenever the income from that choice exceeds the income from choosing wage employment. The individual is assumed to have two separate skills, an assumption that plays a decisive role in the model. The income as entrepreneur will equal the strength of his weakest skill times a parameter reflecting the market value of entrepreneurship talent and the income as employed will correspond to the strength of his strongest skill. It follows that the income distribution among a group of individuals making these choices will reflect the distribution of skill strengths in the group.

The main aim of this paper is to demonstrate that Lazear's model also can be used to analyze the income return for entrepreneurs and wage employees. The income return of an actor is defined as the income resulting from the preferred occupational choice divided by the hypothetical income he or she would earn if for some reason forced to make the opposite choice. We will analyze this by deriving the theoretical implications that follow from the assumption that skill strengths in a population are Fréchet distributed.

Another aim is to shed light on the theoretical results by using data from the Swedish employment register. Defining entrepreneurs as those registered as self-employed, we compute the return to self-employment and wage employment in three parts of Sweden: the Stockholm region, the combined Gothenburg and Malmö region and the Rest of Sweden. In order to obtain some indirect information about the appropriateness of assuming that skill strengths are Fréchet distributed, we will also compare average incomes between self-employed and wage employed as well as their corresponding observed and theoretically derived income distributions.

The calculations are made for all individuals that have a university exam as "Master of science in electrical engineering" and were self-employed or wage employed in 2008. We differentiate between all self-employed and the self-employed that hires at least one person. Both the theoretical derivations and the data draw upon Hårsman and Mattsson (2017).

Disregarding studies that include non-pecuniary benefits to analyze the occupational choice, see for example Benz (2009), our theoretical approach is similar to most others estimating the return to entrepreneurship. The central issue in all of them evolves around the "what-if-question", that is how to estimate the hypothetical incomes entrepreneurs would have earned had they instead chosen to become wage employed. The common approach is to compare the income of entrepreneurs and wage-employed and apply different techniques to sort away the influence of other factors likely to influence the income. Åstebro et al. (2013), for example, compare the current and previous income among Swedish academics that recently have started new businesses making adjustment for income growth over time, whereas Hyytinen et al. (2011) estimate the income differences between identical twins that have made different

occupational choices. Our empirical calculations are different. Using the Lazear model, we compute the parameters of applied Fréchet distributions for several subgroups of our population. Each such calculation rests on only three variables: the relative frequency of self-employment and the average income for self-employed and wage employed.

By definition, the expected return to self-employment and wage employment in Lazear's model cannot fall below one. Taking age, sex or earlier entrepreneurship experience into account, the empirical computations show that the returns to self-employment are meager and much lower than the returns to wage employment in all regions. The return to self-employed, who hire at least one person, is higher in the Stockholm region than in the other two regions – presumably reflecting various agglomeration benefits. However, the regional differences change when also including the self-employed, who do not hire anyone. In the latter case, the self-employed in the Stockholm region have a higher return than those living in the Gothenburg and Malmö region but about the same return as those living in the Rest of Sweden. The calculations do also provide support for the Fréchet assumption. An implication of this assumption is that the average income of self-employed is lower than that of wage employed (see Section 3). With few exceptions, this relationship holds among both types of self-employed in all regions. The similarity between the theoretically derived and empirically observed income distributions for self-employed and wage employed provides additional support for the Fréchet assumption.

The paper is structured as follows. The next section describes the Lazear model and the income resulting from preferred vis-à-vis opposite occupational choices. Section 3 introduces the Fréchet distribution and provides analytical expressions for the expected income returns. Section 4 presents the data and the computed regional income return to entrepreneurship and wage employment. A summary and some conclusions follow in Section 5. Mathematical derivations are collected in Appendix A and some data characterizing our population in Appendix B.

2. Summary of Lazear's model

Lazear (2005) assumes that each individual has two kinds of basic skills, the strength of which are represented by the variables X_1 and X_2, and that the choice to become entrepreneur or wage employee is based upon the resulting income. As wage employed, the income will correspond to the strength of the strongest skill that is $\max(X_1, X_2)$, since employers are expected to hire specialists of different kinds. Working as an entrepreneur, on the other hand, the individual has to use both skills and will receive an income reflecting the strength of his weakest skill, that is $\lambda \min(X_1, X_2)$, where $\lambda > 1$ is the market value of entrepreneurial talent. Any individual will therefore choose entrepreneurship if and only if $\lambda \min(X_1, X_2)$ is larger than $\max(X_1, X_2)$.

Figure 1, adapted from Lazear (2005), can be used to further explain the logic of the model and the consequences for the income of an entrepreneur or a wage employee, who for some reason has to make an occupational choice opposite to the preferred one.

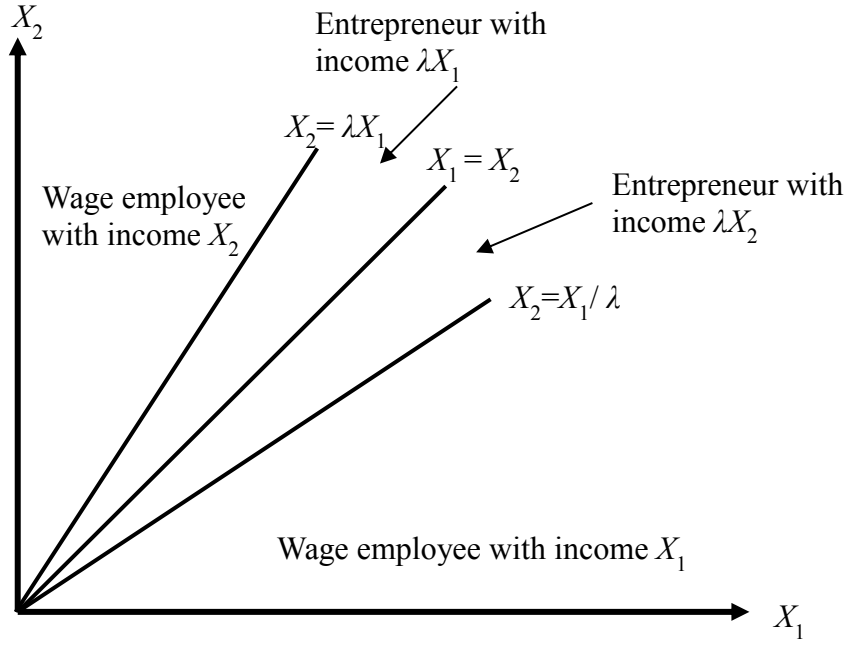

Figure 1. Selection into entrepreneurship or wage employment by individuals endowed with different skill strength vectors (X_1, X_2). Adapted from Lazear (2005)

Individuals having skills of equal strength will be found along the figure's diagonal, those with stronger X_2 than X_1 skill above the diagonal and vice versa. Those located above the line $X_2 = \lambda X_1$ or below the line $X_2 = X_1 / \lambda$ are strong enough in skill X_2 and X_1, respectively, to prefer wage employment and earn the corresponding income. Individuals, who have more balanced skills, the Jacks of All Trades, will choose entrepreneurship and receive incomes equal to λX_1 and λX_2, respectively.

Let us now consider the "what-if-question", that is what income individuals would earn, if they for some reason could not realize their preferred occupational choices. By definition, all would earn less, which implies a positive return to both entrepreneurship and wage employment. The return will vary between individuals having different skill strength vectors. Consider an entrepreneur located above the diagonal but below the line $X_2 = \lambda X_1$. If forced to become wage employed, the income would be equal to the strength of his strongest skill X_2 and his return from entrepreneurship will thus be $\lambda X_1 / X_2$. The return to entrepreneurship for an entrepreneur located below the diagonal but above the line $X_2 = X_1 / \lambda$ will be $\lambda X_2 / X_1$. For corresponding reasons, the "return to wage employment" will equal $X_2 / \lambda X_1$ and $X_1 / \lambda X_2$ for those above the line $X_2 = \lambda X_1$ and below the line $X_2 = X_1 / \lambda$, respectively.

Several conclusions can be drawn from these simple observations. All entrepreneurs will get a higher return, the higher the market value of entrepreneurial talent λ. The individual skill strengths do also matter. The highest possible return λ will be earned only by those whose two skills are perfectly balanced. It will be lower for everyone else and decreases with increased imbalance between the skills, with zero percent return as the lower limit. The opposite applies for individuals preferring to become wage

employed. Their return will be lower the higher the market value of entrepreneurial talent λ, and it will increase with decreased balance between the skills, potentially approaching infinity for a wage employee with a skill vector very close to any of the axes. Since the strengths of individual skills are difficult, if not impossible, to measure, it will be difficult to estimate the income return to the preferred individual choices from direct empirical observations in a population. However, as we will see in the next section, by assuming that the individuals in a population have Fréchet distributed skill vectors, it is possible to measure the expected income return to entrepreneurship and wage employment from observed parameters in the population.

The Lazear model has been rather thoroughly tested. Though the results are mixed, most studies seem to support the basic idea that entrepreneurs have a more balanced skill profile than wage employed. A major criticism is that most empirical studies indicate a negative return to entrepreneurship, a result assumed to be incompatible with the model, and that the model makes no room for non-pecuniary benefits.[4] According to, for example, Åstebro and Thompson (2011) and Hartog et al. (2010), benefits of that kind must be included in order to explain the choice of entrepreneurship in spite of low income returns and long working hours. However, as shown by Hårsman and Mattsson (2017), there is no need to include non-pecuniary benefits for that reason. As previously mentioned, assuming that skill vectors in a population are Fréchet distributed, the lower expected income for entrepreneurs vs. wage employed is fully consistent with Lazear's model.

3. Theoretical analysis of expected income returns

As described in the previous section, each individual is assumed to have two kinds of basic skills, 1 and 2, the strengths of which are represented by the positive values X_1 and X_2 (see Figure 1). We here assume that the skill strengths X_1 and X_2 in a population are random variables, identically and independently distributed according a Fréchet distribution.

A continuous random variable $X > 0$ is Fréchet distributed with shape parameter $\beta > 0$ and scale parameter $v > 0$ if its cumulative distribution function is

$$F(x) = e^{-(v/x)^{\beta}}, \quad x > 0.$$

We here assume that $\beta > 1$. It is well known that the expected value then exists and is $\mathrm{E}[X] = v \cdot \Gamma(1-1/\beta)$.[5] The larger the value of the shape parameter β, the more peaked is the density and the larger the value of the scale parameter v, the larger is the expected value.

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[4] Using Norwegian worker-firm-owner data and extending the definition of self-employment, Berglann et al. (2010) show that the return to entrepreneurs is positive. Åstebro (2017) shows that correcting for underreporting of incomes among Danish self-employed individuals will also result in positive returns.

[5] Γ is the gamma function defined by $\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt$ for $x > 0$.

According to Lazear's model an individual will become entrepreneur if $\lambda \min(X_1, X_2) > \max(X_1, X_2)$, and wage employed if the opposite inequality holds, where $\lambda > 1$ is the market value of entrepreneurial talent. The probability to become entrepreneur, denoted p_E, corresponds to the event that the random vector (X_1, X_2) falls between the lines $X_2 = \lambda X_1$ and $X_2 = X_1 / \lambda$ in Figure 1, and the probability to become wage employed, denoted p_W, to the event that this random vector falls either between the vertical axis and the line $X_2 = \lambda X_1$ or between the line $X_2 = X_1 / \lambda$ and the horizontal axis. It is easily shown that these probabilities are (see Hårsman and Mattsson, 2017):

$$p_E = \frac{\lambda^\beta - 1}{\lambda^\beta + 1},$$

$$p_W = 1 - p_E = \frac{2}{\lambda^\beta + 1}.$$

Let the random variables Y_E and Y_W denote the income of an individual who has chosen to become entrepreneur and wage employed, respectively. Then

$Y_E = \lambda \min(X_1, X_2)$ conditional on the event that $\lambda \min(X_1, X_2) > \max(X_1, X_2)$, and

$Y_W = \max(X_1, X_2)$ conditional on the event that $\max(X_1, X_2) > \lambda \min(X_1, X_2)$.

As previously mentioned, the assumption that skill strengths X_1 and X_2 are identically and independently Fréchet distributed implies that the expected income of an entrepreneur is always lower than that of a wage employed (see Hårsman and Mattsson, 2017, Proposition 3):

$$\mathrm{E}[Y_E] < \mathrm{E}[Y_W].$$

To apply Lazear's model to a population of individuals, we need to determine three parameters from empirical data: λ, β and v. To this end, we identify the share of self-employed in the population with p_E, and the average income of self-employed and wage employed with $\mathrm{E}[Y_E]$ and $\mathrm{E}[Y_W]$, respectively. The above inequality requires that data are such that the average income of self-employed is lower than the average income of wage employed. Let $\varphi = (1 + p_E) / (1 - p_E)$ and $q = \mathrm{E}[Y_E] / \mathrm{E}[Y_W]$. Then (see Hårsman and Mattsson, 2017):

$$\beta = \frac{\ln(2\varphi) - \ln(\varphi + 1)}{\ln(2\varphi - q \cdot (\varphi - 1)) - \ln(\varphi + 1)},$$

$$\lambda = \varphi^{1/\beta}, \text{ and}$$

$$v = \frac{\mathrm{E}[Y_W]}{(1 + \lambda^\beta)^{1/\beta} \cdot \Gamma(1 - 1/\beta)}$$

To define the expected returns to entrepreneurship and wage employment, we first have to define the hypothetical income for an individual, who for some reason has to make an occupational choice opposite to the preferred one. The hypothetical income for an individual, who prefers to become entrepreneur but is forced to become wage employed, is

$$Z_E = \max(X_1, X_2) \text{ conditional on the event that } \lambda \min(X_1, X_2) > \max(X_1, X_2),$$

and the hypothetical income for an individual, who prefers to become wage employed but is forced to become entrepreneur, is

$$Z_W = \lambda \min(X_1, X_2) \text{ conditional on the event that } \max(X_1, X_2) > \lambda \min(X_1, X_2).$$

The expected income return to entrepreneurship is then:

$$\mu_E = \frac{\mathrm{E}[Y_E]}{\mathrm{E}[Z_E]} = \frac{2^{1-1/\beta}\lambda^{\beta} - \lambda(1+\lambda^{\beta})^{1-1/\beta}}{(1+\lambda^{\beta})^{1-1/\beta} - 2^{1-1/\beta}},$$

and the expected income return to wage employment:

$$\mu_W = \frac{\mathrm{E}[Y_W]}{\mathrm{E}[Z_W]} = \frac{(1+\lambda^{\beta})^{1/\beta}}{\lambda(1+\lambda^{\beta}) - \lambda^{\beta}(1+\lambda^{\beta})^{1/\beta}}$$

(see Appendix A for proofs).

Since an entrepreneur who has perfectly balanced skills, that is for whom $X_1 = X_2$, will experience a relative gain of λ by his occupational choice, and one who is indifferent between becoming entrepreneur or wage employed, that is for whom $\lambda X_1 = X_2$ or $\lambda X_2 = X_1$, will experience a relative gain of 1, it must be the case that $1 < \mu_E < \lambda$ and we can expect μ_E to be rather close to $(1+\lambda)/2$.

4. Data and results

In this section we will apply the theoretical derivations to compute and compare the return to self-employment and wage employment for a population of individuals. We will also compare the observed and computed income distributions among self-employed and wage employed.

Our database – provided by Statistics Sweden – comprises all 13,800 self-employed and wage employed individuals in 2008, who have a university exam as "Master of science in electrical engineering" (to be referred to as electrical engineers). Anyone working at least one hour per week in November 2008 is defined as employed, implying that both full-time and part-time workers are included. Each employed individual is categorized as either wage employed or self-employed. Co-owners of a business are included in the self-employed category. Individuals, who combine wage employment and self-employment or co-owning, are classified as wage employees, if their income from wage employment is larger than the income from self-employment or co-owning of a firm and vice versa. Since combinations are rather common – around 10 percent of all kinds of engineers having a master's degree have combined incomes – it should be noted that

even a small income change might result in a reclassification of an individual from say self-employed to wage employed.[6]

One advantage of focusing on a group that is homogenous regarding length and type of education is that both factors are likely to influence the occupational choice and the return to entrepreneurship, see, for example, Berglann et al. (2010) and Poschke (2013). Other reasons for focusing on electrical engineers are that the self-employed among them are more likely to be opportunity-driven than necessity-driven, and that the self-employment rate for this group is higher than among engineers having another type of specialization.[7]

The population of electrical engineers is subdivided into three regions depending on their place of residence: the Stockholm region, the Gothenburg and Malmö region, and the Rest of Sweden.[8] The Gothenburg and Malmö region consists of two geographically separated labor market regions, each of which is much smaller than the Stockholm labor market region in terms of population and employment. About 42 percent of the 13,800 electrical engineers live in the Stockholm region, 38 percent in the Gothenburg and Malmö region and 20 percent in the Rest of Sweden. In relation to the distribution of all working individuals across regions, this implies a strong overrepresentation for the Stockholm and the Gothenburg and Malmö regions; the corresponding shares for all working individuals are 27, 21 and 52 percent, respectively. Although rough, our regional division will hopefully capture the influence of the labor market size on the return to self-employment and wage employment.

The data used as input in our calculations are presented in Table 1, market value of entrepreneurial talent and income return to self-employment and wage employment in Table 2, and shape parameter of the Fréchet distribution and elasticity of self-employment rate with respect to the market value of entrepreneurial talent in Table 3. The section ends with some graphs that compare the observed and computed income distributions for the Stockholm region. Table B1 in Appendix B provides further information about the number of self-employed and wage employed and their average income. The data for each region are subdivided by sex, age and experience. Electrical engineers, who have been self-employed at least two of the preceding four years, are classified as experienced and the rest as having no experience.[9] Two different panels of individuals are considered. In Panel A, all self-employed and all wage employed are included, and in Panel B self-employed who are hiring at least one person together with all wage employed.

The most obvious pattern in Table 1 for both Panel A and Panel B is perhaps that the self-employment rates are higher in the Stockholm region than in the other two regions, and that the rates look rather similar in the Gothenburg and Malmö region and the Rest of Sweden.

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[6] The percentage number is from Daghbashyan and Hårsman (2013).
[7] See Åstebro et al. (2013) and Hårsman and Mattsson (2017), respectively.
[8] In 2008 Sweden was divided into 75 labor market regions defined in terms of commuting between municipalities. Statistics Sweden updates the division regularly.
[9] The variable is rather crude since some of the electrical engineers in 2008 might not have been registered as either self-employed or wage employed during one or more of the preceding years, for example, because of unemployment.

Table 1. Percentage self-employed and ratio between average income of self-employed and of wage employed by region, sex, age and experience of self-employment

Subgroup	Stockholm		Gothenburg and Malmö		Rest of Sweden	
	Percent self-employed	Income ratio	Percent self-employed	Income ratio	Percent self-employed	Income ratio
Panel A: wage employed and all self-employed						
All in Panel A	10.5	0.61	8.2	0.73	8.7	0.52
Male	11.1	0.60	8.8	0.72	9.1	0.50
Female	5.1	0.67	2.8	0.75	3.5	0.94
Less than 45 years	6.8	0.81	4.6	0.87	4.1	0.79
45 or more	14.2	0.50	13.1	0.61	11.6	0.44
No experience	3.6	0.66	2.7	0.78	2.7	0.58
Experience	56.0	0.84	51.3	1.02[a]	50.5	0.69
Panel B: wage employed and all self-employed hiring at least one person						
All in Panel B	4.8	0.72	4.0	0.89	3.7	0.76
Male	5.2	0.70	4.3	0.87	3.9	0.73
Female	1.8	0.84	1.2	0.98	1.5	–[a,b]
Less than 45 years	3.9	0.93	2.6	0.97	1.8	0.91
45 or more	5.9	0.58	6.1	0.77	4.9	0.68
No experience	1.5	0.80	1.1	0.87	0.9	0.92
Experience	36.2	0.98	34.5	1.29[a]	30.5	0.99

[a] Since the income ratio exceeds 1, the subgroup has to be excluded from further analysis
[b] For privacy reason the exact value cannot be reported

The higher self-employment rates in the Stockholm region vis-à-vis the other regions, and among men compare to women, among old compared to young and among those having experience compared to those without experience are in line with earlier studies, see, for example, Bosma and Sternberg (2014), Michelacci and Silva (2007), and Parker (2009). It is more difficult to explain that the self-employment rates are about the same in the Gothenburg and Malmö region and the Rest of Sweden. Since agglomeration benefits of various kinds are likely to stimulate entrepreneurial activities in large urban areas, see, for example, Bosma and Sternberg (2014) and Helsley and Strange (2011), one would expect the self-employment rates to be higher in the Gothenburg and Malmö region. One reason might be that some large manufacturing firms, like for example ABB and mining industries, which are located in the Rest of Sweden, make some labor markets in that region as attractive for self-employed electrical engineers as the

Gothenburg and Malmö region.[10] A related reason might be differential returns to self-employment.

Another common feature is that almost all income ratios are less than 1.[11] That self-employed on average have lower incomes than wage employed is well known from earlier studies and supports our assumption of Fréchet distributed skill profiles. Since an income ratio greater than 1 is inconsistent with our theoretical model, we have to exclude the three subgroups for which this is the case from further analysis. It could be noted that one of them only includes three individuals.

Table 2 presents the computed income returns expressed in percent. The most striking observation is that the returns to self-employment are meager. The average income return μ_E is less than 5 percent for most subgroups of self-employed. We may also note that the percentage income return to self-employment is very close to half of the percentage market value of entrepreneurial talent, as predicted in our theoretical analysis: $\mu_E \approx (1+\lambda)/2$.

Since the occupational choice is driven by the expected returns in the applied theoretical model, one should expect a fairly good correspondence between the calculated returns to self-employment and the self-employment rates reported in Table 1. In line with the expectations, Table 2 shows that the return to self-employment is much higher in the Stockholm region than in the Gothenburg and Malmö region, either one considers the highest possible return λ or the average return μ_E and for both panels. The same advantage for the Stockholm region holds vis-à-vis the Rest of Sweden among the self-employed, who hire at least one person (Panel B). It is more difficult to explain why the return to all kind of self-employed (Panel A) for several subgroups is higher in the Rest of Sweden than in the other two regions and that the self-employed hiring at least one person (Panel B) receive higher returns in the Rest of Sweden than in the Gothenburg and Malmö region. The return differences caused by individual characteristics follow the pattern to be expected from the corresponding pattern of self-employment rates in Table 1.

Table 2 also shows that the average return to wage employment μ_W exceeds the return to self-employment by far in all regions. Agglomeration effects do not seem to explain the regional variation. The returns in the Stockholm region are higher than in the other two regions according to Panel B but not to Panel A and the Gothenburg and Malmö region has lower returns than the Rest of Sweden irrespective of whether the wage employed are compared with all self-employed (Panel A) or only with self-employed hiring at least one person (Panel B).

[10] Out of all workers in manufacturing industries, 19 percent live in the Stockholm region, 20 percent in the Gothenburg and Malmö region, and 61percent in the Rest of Sweden.
[11] Incomes are defined as before tax wage per year for wage employees and before tax net business income per year for self-employed. Statistics Sweden multiplies the reported business income by 1.6 in order to adjust for an observed tendency by business owners to underestimate their business income. The income for those combining wage employment and self-employment includes both types of income.

Table 2. Market value of entrepreneurial talent λ and income return to self-employment μ_E and wage employment μ_W by region, sex, age and experience of self-employment [in percent]

Subgroup	Stockholm			Gothenburg and Malmö			Rest of Sweden		
	λ	μ_E	μ_W	λ	μ_E	μ_W	λ	μ_E	μ_W
Panel A: wage employed and all self-employed									
All in Panel A	8.8	4.3	90	4.7	2.3	52	8.9	4.3	129
Male	9.6	4.7	94	5.3	2.6	56	9.7	4.7	139
Female	3.5	1.8	70	1.4	0.7	46	0.4	0.2	9
Less than 45ys	2.7	1.3	33	1.3	0.6	22	1.8	0.9	38
45 or more	16.1	7.8	144	11.2	5.5	90	14.3	6.9	179
No experience	2.5	1.2	73	1.2	0.6	40	2.3	1.1	101
Experience	28.2	14.0	28	–[a]	–[a]	–[a]	48.6	22.9	64
Panel B: wage employed and all self-employed hiring at least one person									
All in Panel B	2.8	1.4	54	0.9	0.5	18	1.8	0.9	44
Male	3.2	1.6	59	1.2	0.6	21	2.1	1.1	52
Female	0.6	0.3	27	0.1	0.0	3	–[a]	–[a]	–[a]
Less than 45 ys	0.6	0.3	11	0.2	0.1	5	0.3	0.2	15
45 or more	5.2	2.6	103	2.9	1.4	41	3.2	1.6	65
No experience	0.6	0.3	35	0.3	0.1	22	0.1	0.1	12
Experience	1.9	1.0	3	–[a]	–[a]	–[a]	0.7	0.4	1

[a] Since the income ratio exceeds 1, the subgroup has been excluded from analysis

At a general level, differential regional returns to self-employment can be explained by differential demand or by shifts of the supply curve, shifts that may be caused by agglomeration effects, see, for example, Glaeser et al. (2009). The β-parameter in our model plays a role corresponding to a regional shift variable, as illustrated in Figure 2. The supply curves in the figure relate the probability of being entrepreneur to the market value of entrepreneurial talent; movements along a given supply curve reflects demand changes and the supply curve shifts with the value of the β-parameter.

The figure shows that the probability to become entrepreneur p_E increases with λ along each supply curve, and that the supply curve shifts out when the value of β increases. It also seems as if the elasticity of p_E with respect to λ decreases with increased values of λ. The elasticity, given by the following expression

$$\varepsilon_{p_E,\lambda} = \frac{\partial p_E / \partial \lambda}{p_E / \lambda} = \frac{2\beta \cdot \lambda^{\beta}}{\lambda^{2\beta} - 1},$$

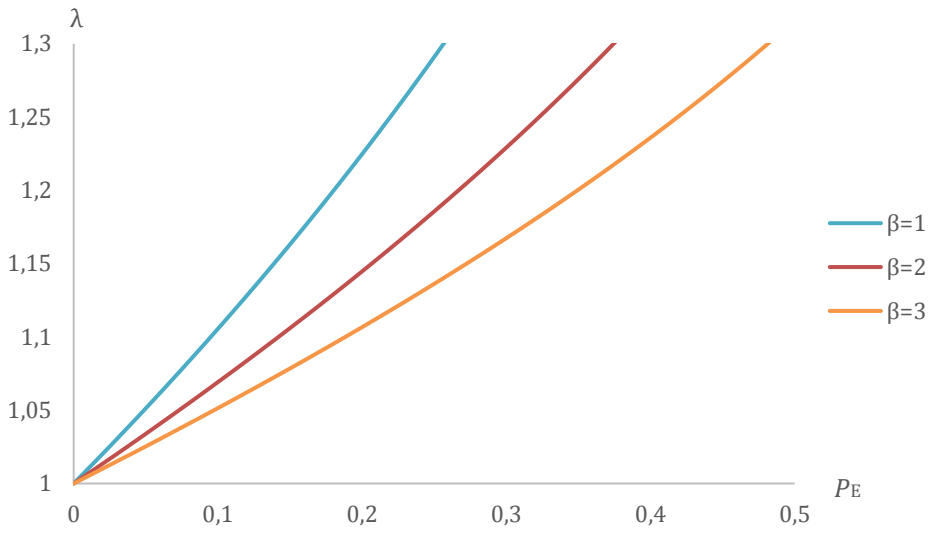


Figure 2. Shifts of the of the entrepreneurial supply curve resulting from changes of the Fréchet shape parameter β

is positive, and it is easily be confirmed that its derivative with respect to λ is negative.

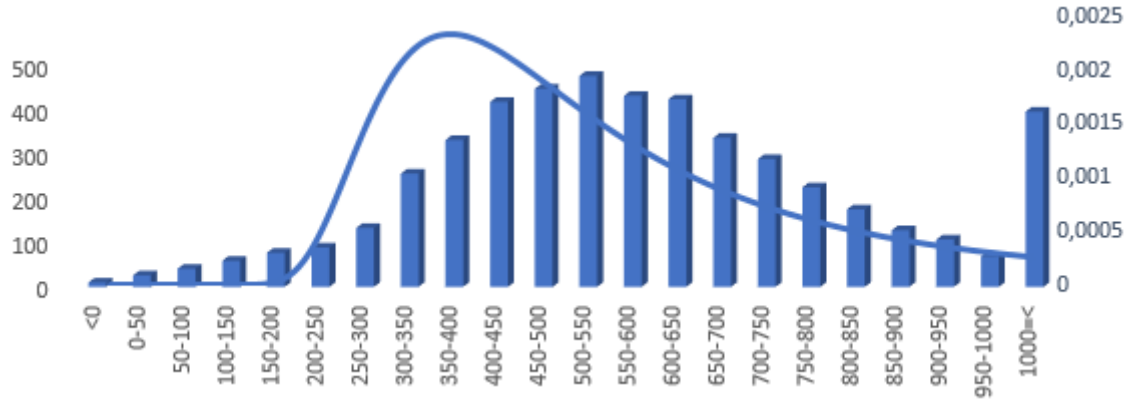
Table 3 shows that the shape of the skill strength distributions, as measured by the β-parameter, differs between regions as well as between the subgroups defined by sex, age and earlier entrepreneurship experience. With few exceptions, the β-values are higher in the Gothenburg and Malmö region than in the other regions according to both panels. This might explain why the rates of self-employed are relatively high in the Gothenburg and Malmö region in spite of relatively low λ-values as reported in Tables 1 and 2. That the β-values are higher in the Stockholm region than in the Rest of Sweden in Panel A but the order is reversed in Panel B, might in a similar way explain the much higher self-employment rates in the Stockholm region despite the fact that the λ-values are about the same in the two regions.

The elasticities are high and vary greatly between both regions and subgroups. The high values reported for the Gothenburg and Malmö region imply that even a small increase of the market value of entrepreneurial talent would increase the entrepreneurship rate substantially. A comparison with Table 2 shows that higher λ-values imply lower elasticities as indicated by Figure 2.

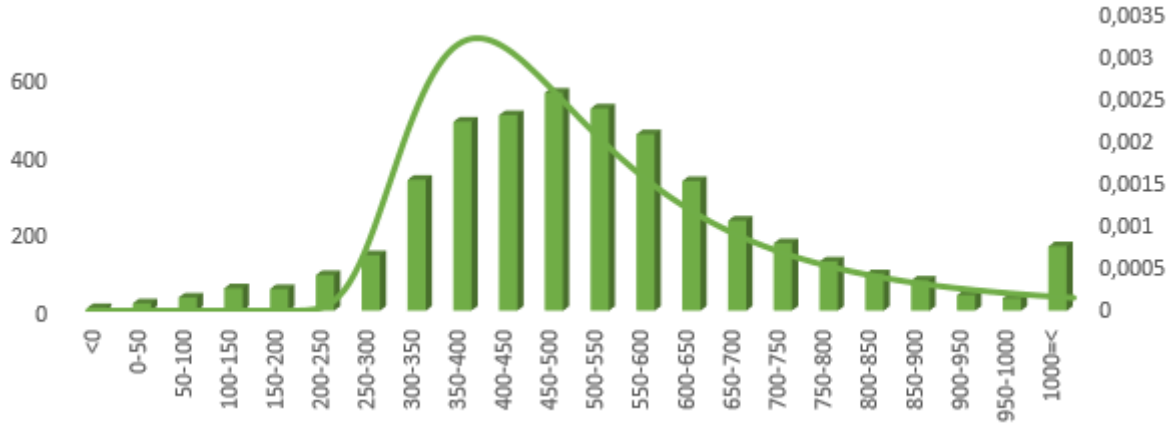
Although most of the income ratios shown in Table 1 support our assumption that the skill strengths are Fréchet distributed, more "evidence" is certainly needed to confirm this assumption. The comparison of the observed and computed income distributions in Figure 3 provides some additional support. All comparisons relate to the Stockholm region. The (a) part shows the observed and computed income distributions for wage employees, the (b) and (c) part the same for all self-employed and the self-employed hiring at least one person, respectively.

The equality between the distributions is quite good for the (a) and (b) part. It is far from good for the (c) part – the computed distribution is skewed to the left, while the observed one rather is slightly right-skewed.

(a) All wage employed



(b) All self-employed



(c) All self-employed hiring at least one person

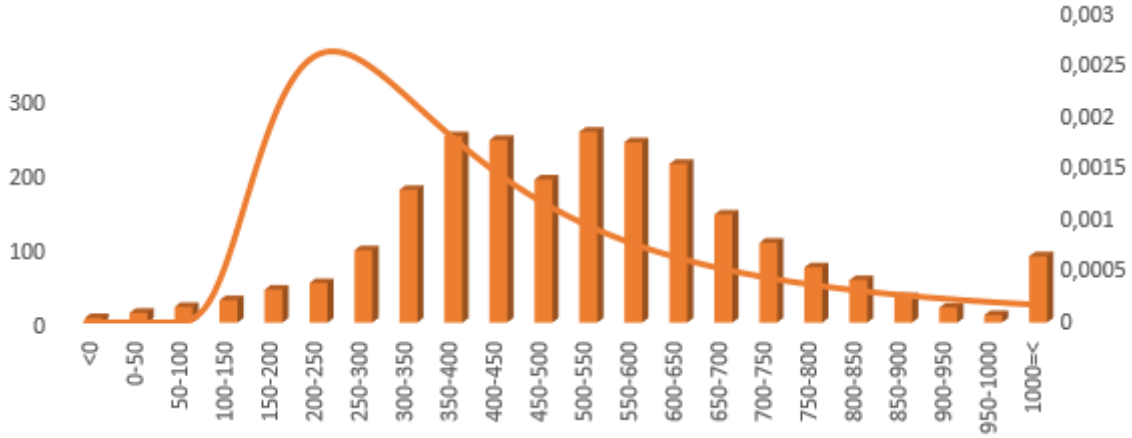


Figure 3. Observed and computed income distributions in the Stockholm region

Table 3. Shape parameter of the Fréchet distribution β and elasticity of self-employment rate with respect to market value of entrepreneurial talent $\varepsilon_{P_E,\lambda}$, by region, sex, age and experience of self-employment

Subgroup	Stockholm		Gothenburg and Malmö		Rest of Sweden	
	β	$\varepsilon_{P_E,\lambda}$	β	$\varepsilon_{P_E,\lambda}$	β	$\varepsilon_{P_E,\lambda}$
All in Panel A	2.5	12	3.6	22	2.0	12
Male	2.4	11	3.4	19	2.0	11
Female	3.0	29	4.0	71	15.5	225
Less than 45ys	5.1	38	7.3	80	4.6	57
45 or more	1.9	7	2.5	9	1.7	7
No experience	2.9	41	4.4	83	2.4	44
Experience	5.1	3	–[a]	–[a]	2.8	2
All in Panel B	3.5	36	8.6	107	4.1	56
Male	3.3	32	7.4	85	3.7	47
Female	6.1	167	49.0	1994	–[a]	–[a]
Less than 45 ys	13.5	175	28.5	554	10.4	286
45 or more	2.3	20	4.3	35	3.1	31
No experience	4.9	166	7.4	348	12.7	708
Experience	41.0	49	–[a]	–[a]	87.4	130

[a] Since the income ratio exceeds 1, the subgroup has been excluded from analysis

5. Summary and conclusions

The theoretical part of the paper investigates the return to entrepreneurship by combining Lazear's model for occupational choice with the assumption that skill strengths are Fréchet distributed. The income return of an individual is defined as the income resulting from the preferred occupational choice divided by the hypothetical income he or she would earn if for some reason forced to make the opposite choice. The two analytical expressions derived demonstrate the extent to which the expected income from the preferred choice exceeds the expected income from the opposite choice for entrepreneurs as well as for wage employed. As earlier shown by Hårsman and Mattsson (2017), the parameters included in the two expressions can be computed by using commonly available data on occupational choice and average incomes for entrepreneurs (self-employed) and wage employed.

In the empirical part of the paper, we use data from the Swedish employment register to compute the return to self-employment and wage employment in three parts of Sweden: the Stockholm region, the Gothenburg and Malmö region and the Rest of Sweden. The calculations are made for all individuals that have a university exam as "Master of

science in electrical engineering" and were self-employed or wage employed in 2008. We differentiate between all self-employed and self-employed hiring at least one person and include sex, age and earlier self-employment experience as criteria for dividing into subgroups.

Our computations show that the average returns to entrepreneurship are less than 5 percent for most subgroups of self-employed and the return to wage employment over 50 percent in more than half of the wage employment subgroups. They also show that the return to the self-employed hiring at least one person is higher in the Stockholm than in the other two regions. All kinds of self-employed have higher returns if they are males, 45 years or older or if they have earlier experience of self-employment. The same holds for the income returns to wage employment for all subgroups except for those with earlier experience of self-employment. More generally, our calculations demonstrate the need to differentiate between self-employed who only employ themselves and those who also hire other persons.

According to our data the ratio between average income of self-employed and wage employed is less than 1 for almost all subgroups consistent with our theoretical model. Together with the similarities between the computed and observed income distributions for self-employed and wage employed, this provides support for the Fréchet assumption in our application of the Lazear model.

In spite of the empirical support for the Fréchet distribution, it would be interesting to test other skill strength distributions, and to derive the implications of relaxing the assumption that different skills are independently distributed across a population as well as allowing more than two different skills.

The empirical calculations demonstrate the importance of considering the differential regional conditions for entrepreneurship. It would, for example, be interesting to compute the income return to self-employment for a panel of labor markets and relate it characteristics like population, industrial mix and accessibility.

Appendix A. Proof of propositions

A random variable X is said to be Fréchet(β, v), if it is Fréchet distributed with shape parameter $\beta > 0$ and scale parameter $v > 0$, i.e. if its cumulative distribution function is $F(x) = e^{-(v/x)^{\beta}}, \quad x > 0.$ Trivially, if X is Fréchet(β, v), then αX is Fréchet($\beta, \alpha v$) for $\alpha > 0$.

Proposition 1. Properties of the Fréchet distribution

Let two random variables X_1 and X_2 be independently $\text{Fréchet}(\beta, v_1)$ and $\text{Fréchet}(\beta, v_2)$, respectively, with $\beta > 1$. Then

$$p_1 = \mathrm{P}(X_1 > X_2) = \frac{v_1^{\beta}}{v_1^{\beta} + v_2^{\beta}},$$

$$p_2 = \mathrm{P}(X_2 > X_1) = \frac{v_2^{\beta}}{v_1^{\beta} + v_2^{\beta}}.$$

Moreover,

$\hat{X} = \max(X_1, X_2), \hat{X}_1 = (X_1 \,|\, X_1 > X_2)$ and $\hat{X}_2 = (X_2 \,|\, X_2 > X_1)$ are all $\text{Fréchet}(\beta, \hat{v})$,

with the same expected values

$$\mathrm{E}[\hat{X}] = \mathrm{E}[\hat{X}_1] = \mathrm{E}[\hat{X}_2] = \hat{v} \cdot \Gamma(1 - 1/\beta),$$

where

$$\hat{v} = (v_1^{\beta} + v_2^{\beta})^{1/\beta}.$$

Proof. All claims follow immediately from e.g. Corollary 3 in Mattsson et al. (2014).

Proposition 2. Expected income return to entrepreneurship

Expected income return to entrepreneurship is

$$\mu_E = \frac{\mathrm{E}[Y_E]}{\mathrm{E}[Z_E]} = \frac{2^{1-1/\beta}\lambda^{\beta} - \lambda \cdot (1+\lambda^{\beta})^{1-1/\beta}}{(1+\lambda^{\beta})^{1-1/\beta} - 2^{1-1/\beta}}.$$

Proof. By Hårsman and Mattsson (2017, Proposition 2)

$$\mathrm{E}[Y_E] = \frac{2\lambda^{\beta} \cdot (1+\lambda^{\beta})^{1/\beta} - 2^{1/\beta}\lambda \cdot (1+\lambda^{\beta})}{\lambda^{\beta} - 1} \cdot v \cdot \Gamma(1 - 1/\beta).$$

Recall that the random skill strengths X_1 and X_2 are identically and independently $\text{Fréchet}(\beta, v)$. By the law of total expectation

$$\mathrm{P}(X_2 > X_1) \cdot \mathrm{E}[(X_2 \,|\, X_2 > X_1)] = \mathrm{P}(X_2 > \lambda X_1) \cdot \mathrm{E}[(X_2 \,|\, X_2 > \lambda X_1)]$$

$$+ \mathrm{P}(\lambda X_1 > X_2 > X_1) \cdot \mathrm{E}[(X_2 \,|\, \lambda X_1 > X_2 > X_1)].$$

Since $\mathrm{P}(X_2 > X_1) = 1/2$, $\mathrm{P}(X_2 > \lambda X_1) = p_W / 2$, $\mathrm{P}(\lambda X_1 > X_2 > X_1) = p_E / 2$ and $\mathrm{E}[(X_2 \,|\, \lambda X_1 > X_2 > X_1)] = \mathrm{E}[Z_E]$, we have

$$\mathrm{E}[Z_E] = \frac{\mathrm{E}[(X_2 \,|\, X_2 > X_1)] - p_W \,\mathrm{E}[(X_2 \,|\, X_2 > \lambda X_1)]}{p_E}.$$

Since $\mathrm{E}[(X_2 \,|\, X_2 > X_1)] = 2^{1/\beta} v \cdot \Gamma(1-1/\beta)$ and $\mathrm{E}[(X_2 \,|\, X_2 > \lambda X_1)] = (1+\lambda^\beta)^{1/\beta} \cdot v \cdot \Gamma(1-1/\beta)$ by Proposition 1, we have

$$\mathrm{E}[Z_E] = \frac{2^{1/\beta} - \dfrac{2}{1+\lambda^\beta} \cdot (1+\lambda^\beta)^{1/\beta}}{\dfrac{\lambda^\beta - 1}{\lambda^\beta + 1}} \cdot v \cdot \Gamma(1-1/\beta)$$

$$= \frac{2^{1/\beta}(1+\lambda^\beta) - 2 \cdot (1+\lambda^\beta)^{1/\beta}}{\lambda^\beta - 1} \cdot v \cdot \Gamma(1-1/\beta)$$

from which the proposition follows.

Proposition 3. Expected income return to wage employment
Expected income return to wage employment is

$$\mu_W = \frac{\mathrm{E}[Y_W]}{\mathrm{E}[Z_W]} = \frac{(1+\lambda^\beta)^{1/\beta}}{\lambda \cdot (1+\lambda^\beta) - \lambda^\beta \cdot (1+\lambda^\beta)^{1/\beta}}.$$

Proof. By Proposition 1

$$\mathrm{E}[Y_W] = \mathrm{E}[(X_2 \,|\, X_2 > \lambda X_1)] = (1+\lambda^\beta)^{1/\beta} \cdot v \cdot \Gamma(1-1/\beta)$$

as in the proof of Proposition 2. By the law of total expectation and Proposition 1

$$\mathrm{E}[\lambda X_2] = \mathrm{P}(\lambda X_2 < X_1) \cdot \mathrm{E}[(\lambda X_2 \,|\, \lambda X_2 < X_1)] + \mathrm{P}(\lambda X_2 > X_1) \cdot \mathrm{E}[(\lambda X_2 \,|\, \lambda X_2 > X_1)]$$

$$= \frac{1}{1+\lambda^\beta} \cdot \mathrm{E}[Z_W] + \frac{\lambda^\beta}{1+\lambda^\beta} \cdot (1+\lambda^\beta)^{1/\beta} \cdot v \cdot \Gamma(1-1/\beta).$$

Since $\mathrm{E}[\lambda X_2] = \lambda \cdot v \cdot \Gamma(1-1/\beta)$ by Proposition 1, we have

$$\mathrm{E}[Z_W] = \left(\lambda \cdot (1+\lambda^\beta) - \lambda^\beta \cdot (1+\lambda^\beta)^{1/\beta}\right) \cdot v \cdot \Gamma(1-1/\beta)$$

from which the proposition follows.

Appendix B

Table B1. Number of individuals and average annual income of each subgroup

Subgroup	Stockholm		Gothenburg and Malmö		Rest of Sweden	
	Number of individuals	Average annual income [kSEK]	Number of individuals	Average annual income [kSEK]	Number of individuals	Average annual income [kSEK]
Wage employed						
All	4,948	630	4,552	534	2,377	545
Male	4,413	648	4,069	548	2,181	559
Female	535	474	483	414	196	391
Less than 45 years	2,597	550	2,724	473	969	448
45 or more	2,351	717	1,828	625	1,408	611
No experience	4,628	643	4,278	544	2,215	556
Experience	320	440	274	368	162	392
Self-employed						
All	579	385	406	390	226	285
Male	550	388	392	393	219	282
Female	29	317	14	311	7	366
Less than 45 years	189	446	131	409	41	352
45 or more	390	355	275	381	185	270
No experience	171	423	117	423	61	324
Experience	408	369	289	377	165	270
Self-employed hiring at least one person						
All	252	454	190	473	91	415
Male	242	456	184	475	88	408
Female	10	398	6	405	3	–[a]
Less than 45 years	104	510	72	456	18	406
45 or more	148	414	118	483	73	418
No experience	70	513	46	471	20	512
Experience	182	431	144	473	71	388

[a] For privacy reason no income value can be reported due to too few observations

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