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The Impact of ESG on Stocks' Downside Risk and Risk Adjusted Return*

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Abstract

Investments considering corporate social responsibility continue to expand. Are companies pursuing a CSR agenda benefiting shareholders by reducing their financial downside risk? This paper investigates the relationship between a firm's environmental, social and corporate governance (ESG) scores and its downside risk on the stock market. We study this link using a panel of 887 stocks listed in five European countries over the period 2005-2017. Our empirical results show that higher ESG scores are associated with reduced downside risk of stock returns. Based on the Fama-French three factor model, we found no systematic relationship between ESG and the level of risk-adjusted return.

Key Words: ESG; Value at Risk; Risk-adjusted return, stock market; panel data
JEL codes: D22, G11, G14; G32

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1 Introduction

Minimizing the risk of financial investments is an important part of investors' decisions. Asset managers increasingly consider corporate social responsibility (CSR) as a safeguard for limiting the downside risk of their investment. This development is explained by a vast literature showing that society's demands for corporate social responsibility as alternative responses to market and distributive failures are becoming increasingly prominent (Bénabou & Tirole 2010).

For asset managers the application of ESG measures to reflect corporate social performance (CSP) has received a growing attention and is currently demanded by most financial investors. In the U.S., more than a quarter of total assets under professional management (AUM) are using so-called socially responsible investment strategies (USSIF 2018). The corresponding figure for Europe is more than 50% of professionally managed assets (EUROSIF 2016). Furthermore, the members of the United Nations-supported Principles for Responsible Investment(UN PRI) initiative use ESG ratings to assess the social responsibility of investments.

While a number of meta-analyses suggest a positive association between CSR and firms' financial performance, fewer studies exist on the link between firms' social policy and its financial downside risk. The closest work to ours is Hoepner et al. (2018), which exploits yearly data on 1,131 firms and find that environmental, social and corporate governance (ESG)-engagements are associated with lower downside risk, mainly explained by governance improvements.

Using the daily historical returns of 887 stocks listed on five European stock markets over the period 2005 to 2017, we compute Value at Risk (VaR_θ) estimates for each stock from 2009 to 2017 by using a backtesting process. We set a moving window of 1000 trading days in which we employ autoregressive time-series techniques to obtain the necessary 1-day forecasts of stock returns and variance for computing the VaR for each stock. The Var estimate is employed to quantify downside risk, which is a commonly-used measure in asset management to express financial risk.

Thus, the purpose of our research is to test whether the (ESG) score and its individual pillars (ENV, SOC, GOV) have an impact on stock returns' downside risk. We empirically test the hypothesis that an inverse relation between ESG performance and stocks' Value-at-Risk (VaR) exists. The empirical analysis also includes a Fama–French three factor model with the a priori assumption that successful CSP engagement, reduced downside volatility and lower downside risk should not be positively associated with a high risk-adjusted return.

There are several motivations for our focus on European countries in the study. The EU has passed various directives to mitigate climate change such as the Emissions Trading Scheme, which puts a yearly cap on greenhouse gas emissions that is lowered every year, the directive on non-financial information disclosure in the management report (European Commission 2013), and the Sustainability Act[1] which requires that all companies above a certain size establish an annual sustainability report with information on the environment, social conditions, staff, respect for human rights and avoidance of corruption. European companies are considered to be world leaders in CSP (Ho et al. 2012). Europe is also leading the world when it comes to implementing the Paris agreement'S commitment to keeping global warming to well below 2 degrees C. There is also a growing interest among European investors in directing funding towards sustainable actions. Such impact investment increased fivefold between 2013 and 2017 (Eurosif 2018)[2]. However, it should also be noted that considerable heterogeneity on social investing exists within the EU.

Why should we expect companies pursuing a CSR agenda benefiting shareholders by reducing their financial downside risk? The theoretical literature on corporate objectives, profit-making and damage-generating activities, market value and stakeholder welfare is still thin. Milton Friedman Friedman (1970) argued that the goal of a corporation is to maximize the firm's profit and thus to maximize the returns for its shareholders. Shifting the focus to ESG-related activities might increase costs, thereby reducing share price and accordingly harming the shareholders. The existing literature on social investing claims that Friedman is right only if the strong Arrow–Debreu type of conditions holds where each firm is a perfect competitor, full information exists, there is no market uncertainty,

[1]The Sustainability Act entails the implementation of the European Parliament and Council of Europe Directive 2014/95 / EU, (the Sustainability Directive) and applies from 2017 in most member countries. The company's auditor must check whether a sustainability report has been prepared.

[2]A group of European-domiciled sustainable funds that incorporate ESG-factors outperformed the overall fund universe in 2018. https://www.morningstar.com/blog/2019/02/07/european-esg-funds.html

and the government is able to perfectly internalize externalities through laws and regulation. However, the existing literature on social investing has convincingly shows that this is not the case in the real world economy (see for instance Hong & Kacperczyk 2009, Elhauge 2005, Bénabou & Tirole 2010).

In a recent study, Hart & Zingales (2017) argue that a main problem with Friedman's enormously influential article among both economists and lawyers is its narrow view of firm objectives. In line with Freeman et al. (2004) and Freeman (2010), the authors claim that in order to be successful, a firm has to care about the value creation for a wide set of stakeholders including employees, suppliers, customers, financiers, public interest groups, and governmental bodies. This perspective is also formulated by the European Commission which defines CSR as the responsibility of enterprises for their impact on society to integrate social, environmental, ethical, human rights and consumer concerns into their business operations and core strategy in close collaboration with their stakeholders (Commission et al. 2011).

A number of meta-analyses suggest a positive association between CSR and corporate financial performance (see for example Orlitzky et al. 2003, Eccles et al. 2014, Goss & Roberts 2011). A high ESG score signals to investors that the firm is well managed and has good governance structures in place. Following principles of corporate social responsibility practices, firms with high ESG scores are assumed to be less likely to be involved in scandals. Hence this should reduce the downside risk of financial returns.

Many existing econometric studies on ESG and various measure of firm performance suffer from minimal time variation in the measure of firms' social policy due to limited observation periods and mainly yearly observations. This paper extends ESG analysis by exploiting daily stock market returns and monthly ESG scores over a period of 12 years for nearly 900 European stocks. In addition to analysing the effects of ESG on VaR, we also apply the Fama–French three factor model (Fama & French (1992)) to test whether changes of ESG scores affect risk-adjusted returns.

The empirical findings of our econometric analysis reveal that firms with increased ESG scores have lower financial downside risks as described by VaR. An important implication of reduced downside risk is that firms can lower their capital costs, not only on equity markets but also with respect to debt. For instance, a bank might be willing to give a firm a loan with lower interest rates if that firm's ESG scores are high and thus the firm-specific risk is low. In the equity market, a growing number of investment funds consider ESG factors in their investment strategies, which may lead to an increased focus on companies that are managing environmental and social issues effectively and have strong corporate governance practices. As our study shows, these firms tend to be lower-volatility and presumably higher-quality companies that fare better during downturns.

In contrast to numerous previous studies, the analysis reveals no relationship between changes in ESG scores and a stock's risk-adjusted return. However, our results are in line with the capital asset pricing model (CAPM), which is commonly used to determine a theoretically required rate of return of an asset in order to make decisions about adding assets to a well-diversified portfolio. If a firm has high risk, a higher return is required as compensation. As risk can also be measured by volatility, it is expected that VaR and stock return volatility are correlated. Thus, firms with lower downside risk should have lower required returns in equilibrium. However, changes in ESG scores might already have been anticipated by investors and do not lead to surprises. Furthermore, it could be that asset managers rebalance their portfolios in the long-term with respect to ESG, but do not take short-run changes of ESG into account.

The rest of the paper is organized as follows. Section 2 presents the empirical methodology. Data and descriptive statistics are revealed in Section 3, while the empirical results are provided in Section 4. The final section concludes.

2 Empirical methodology

2.1 Value-at-Risk and ESG score

We use Value at Risk (VaR) as a metric for financial risk and investigate whether the ESG score or the scores of its individual pillars (E, S, G) are reducing VaR, and thus reduce a stock's downside risk. We examine this link by using panel data regressions specified as

$$VaR_{\theta i,t} = \gamma_0 + \gamma_i + \gamma_1 \Delta ESG_{i,t} + \gamma_2 \Delta ESG_{i,t-1} + \gamma_3 \Delta ESG_{i,t-2} + \gamma_4 \Delta ESG_{i,t-3} + e_{i,t} \quad (1)$$

where $VaR_{\theta i,t}$ is VaR at level θ% of stock i in month t, γ_i is a stock-specific effect, and $\Delta ESG_{i,t}$ denotes the first difference of $\log(ESG_{i,t}/ESG_{i,t-1})$. We also include three lagged values of $\Delta ESG_{i,t}$.

In order to obtain the dependent variable of the model, $VaR_{\theta i,t}$, we estimate four types of VaR for each firm based respectively on a conditional and an unconditional GARCH model and on estimates from the unconditional and conditional EVT model. In both approaches GARCH and EVT we assume Student t innovations. Finally, we apply the Christoffersen test (Christoffersen 1998) to take into account the VaR models which passed the test and to select which VaR model out of the four alternatives best fits each firm.

Using the daily historical stock returns of 887 firms (81 listed in Sweden and 796 listed in four other European countries) from 2005-08-30 until 2009-06-29, we compute VaR estimates for each stock from 2009-06-30 to 2017-12-29 through a backtesting process. We set a moving window of 1000 trading days (approximately four years) in which we employed an AR(1) and a GARCH(1,1) model with Student t distributed residuals to obtain the necessary one-day-ahead forecasts of stock returns and variance for computing the VaR for each model.

Regarding the two VaR models using the GARCH approach, $VaR_\theta(1000days)$ is computed using the equation:

$$VaR_\theta = -min(\widehat{r(1)} - t_v^{-1}(\theta)\widehat{\sigma(1)}; 0) \tag{2}$$

where θ is the VaR level (95%,99%, or 99.5%), t_v^{-1} (θ) is given by Pr[z< t_v^{-1} (θ)]= θ with standardized $t \sim t_v(0,1)$, $\widehat{r(1)}$ is the conditional Eq. (3) or unconditional Eq. (4) estimated return forecast through the AR(1) process, and $\widehat{\sigma(1)}$ is the conditional (5) or unconditional (6) estimated standard deviation forecast through the GARCH(1,1) process.

$$\hat{r}_{t,cond} = \varphi_0 + \varphi_1 r_{t-1} \tag{3}$$

$$\hat{r}_{unc} = \frac{\varphi_0}{1-\varphi_1} \tag{4}$$

$$\hat{\sigma}^2_{t,cond} = w + \alpha\varepsilon^2_{t-1} + \beta\sigma^2_{t-1} \tag{5}$$

$$\hat{\sigma}^2_{unc} = \frac{w}{1-\alpha-\beta} \tag{6}$$

The restrictions $w > 0, \alpha \geq 0, \beta \geq 0$ and $\alpha + \beta < 1$ are imposed to ensure a positive volatility and a covariance stationary process. ε_t denotes the error term with respect to the AR(1) process in Eq. (3). For Eq. (5), $\varepsilon_t = \sigma_t z_t$ and $z_t \sim iid\, t_v(0,1)$.

In terms of the Extreme Value Theory, we apply the Tail approach setting a threshold of 10% for each moving window (1000 observations). As we deal with financial losses and therefore with the left tail of the empirical return distribution of each stock, we consider the 100 lowest returns included in each rolling window for each stock. We then subtract the lowest value of these returns from each value of the same subset, obtaining an excess distribution which according to the theorem of Balkema & De Haan (1974), and Pickands III et al. (1975) converges to the Generalised Pareto Distribution (GPD) denoted by $G_{\xi,\Psi,(u)}(x)$ as the threshold increases, with

$$F_u(x) \rightarrow G_{\xi,\Psi,(u)}(x) \quad \text{as u } \rightarrow \infty \tag{7}$$

$$G_{\xi,\psi}(x) = \begin{cases} 1-(1+\xi x/\Psi)^{-1/\xi} & \text{if } \xi \neq 0 \\ 1-exp(-x/\Psi) & \text{if } \xi = 0 \end{cases} \quad \psi > 0 \tag{8}$$

where ξ is the tail index, ψ is a scale parameter which specify the distribution and u the threshold, $x \geq 0$ if $\xi \geq 0$ and $0 \leq x \leq -\psi/\xi$ if $\xi < 0$.

The parameter vector (ξ, ψ) of the GPD distribution is estimated by maximizing the log-likelihood:

$$l_t(\xi,\Psi) = \begin{cases} -(\frac{1}{\xi}+1) \quad log(\psi+\xi(x_t-u)) & \text{if } \xi \neq 0, \Psi+\xi(x_t-u) > 0 \\ -log(\psi)-(x_t-u)/\psi & \text{if } \xi = 0 \end{cases} \tag{9}$$

Finally, the associated quantiles which express the unconditional VaR using the EVT tail approach are calculated by the following relations:

$$\hat{q}_{\theta} = u + \frac{\Psi}{\xi}\left[\left(\frac{T}{N_u}(1-\theta)\right)^{-\xi} - 1\right] \tag{10}$$

$$\hat{q}_{uncEVT,\theta} = VaR_{\theta}. \tag{11}$$

With respect to the Conditional VaR using the EVT tail approach, we apply the method suggested by McNeil & Frey (2000), which incorporates the AR-GARCH type model to the EVT. Specifically, the previous AR(1) and GARCH(1,1) dynamics are used in order to account for serial correlation and heteroscedasticity, providing approximately i.i.d. standardized residuals. The parameters of the GPD are estimated for the tails of the distribution of these standardized residuals. We obtain a quantile that belongs to the $t_v(0,1)$ distribution based on the residual distribution assumption. Finally, the conditional VaR with EVT (Eq. (13)) is similar to that in the first part where the conditional and unconditional VaR through the AR-GARCH approach is considered in Eq. (2). The only difference is that the quantile is now that estimated with the EVT tail approach. In other words, the previously described EVT technique is implemented on the standardized residuals Eq. (12) obtained from the AR(1)-GARCH(1,1) models.

$$\hat{Z}_t = (r_t - \hat{r}_{t,cond})/\hat{\sigma}_{t,cond} \tag{12}$$

$$VaR_{condEVT,\theta} = -min\left(\hat{r}_{t,cond} - \hat{q}_{condEVT,\theta^{\hat{\sigma}_{t,cond}}}; 0\right) \tag{13}$$

where r_t is the actual return at time t, $\hat{r}_t$ is the estimated return through the AR(1) process, and $\hat{\sigma}_t$ is the estimated standard deviation through the GARCH(1,1) process.

2.2 VaR model selection

After the computation of VaR for each stock based on the four methods described above, we implement the Christoffersen test which requires the calculation of the number of violations occurred in each method. This represents the number of times when the actual return loss was higher than that predicted by each model. After this step we apply the two partial tests that the Christoffersen test contains: the unconditional coverage test and the test of independence, known as conditional coverage test (CC). The unconditional coverage test evaluates the coverage of the VaR estimates. According to the unconditional coverage property, the probability of facing a loss not exceeding $VaR(\theta)$ must be approximately $(1-\theta)$.

For instance, if a confidence level of $\theta = 99\%$ is used, the null hypothesis is that the frequency of tail losses is equal to $p = 100 - \theta\%$. Assuming that the model is accurate, the observed failure rate x/T should act as an unbiased measure of p, and thus converge to 1% as the sample size increases. On the other hand, the test of independence accounts for any clustering of violated VaR estimates as a model may exhibit dependent VaR estimates (Campbell et al. 2005), implying that the values of the indicator function could be clustered over time. Hence, the CC test largely depends on the frequency of consecutive exceedances.

Finally, the joint test of coverage and independence, which corresponds to the test of conditional coverage, is given by the test statistic $LRcc$. The $LRcc$ is asymptotically distributed as a $\chi^2(2)$ under the null hypothesis.

$$LR_{cc} = LR_{unc} + LR_{ind} \tag{14}$$

After obtaining the $LRcc$ value for each method per company and taking into account only the VaR_θ estimates which passed the test, we select the VaR model with the lowest $LRcc$ value to represent each stock's VaR_θ.

2.3 Fama–French three-factor model for return prediction including an ESG factor

We use the Fama & French (1992) three-factor model adding the sustainability factors (either ESG, ENv, SOC, or GOV score) as an extra factor in order to observe whether a sustainability metric can provide an additional explanation of the stock returns' movements. The model is estimated as:

$$r_{i,t} - r_{f,t} = \delta_0 + \delta_1 \left(r_{m,t} - r_{f,t}\right) + \delta_2 SMB_{i,t} + \delta_3 HML_{i,t} + \delta_4 \Delta ESG_{i,t} + u_i + \varepsilon_{i,t} \tag{15}$$

where $r_{i,t} - r_{f,t}$ is the excess return of the each firm's stock return over the risk-free interest rate, $(r_{m,t} - r_{f,t})$ is the excess return of the market portfolio, $SMB_{i,t}$ is the Small minus Big Factor which is the difference between the return on a portfolio of firms with a low market value of equity (Small cap) and the return on a portfolio of firms with a high market equity value (Large cap), $HML_{i,t}$ is the High minus low (HML) factor which is the difference between the return on a portfolio of firms with a high book-to market value (value stocks) and the return on a portfolio of firms with a low book-to-market value (growth stocks), $\Delta \log ESG_{i,t}$ is the first difference of one of the aforementioned sustainability factors, u_i is a stock-specific effect and $\varepsilon_{i,t}$ represents the residual. The coefficients are estimated using a panel fixed effects regression model. As the Fama–French factors can be interpreted as captured different types of risks that have an impact on the stock's return, the dependent variable is interpreted as the risk-adjusted return.

Note that as we focus on monthly stock returns, any effect from ΔESG implies a short-term change of ESG value. In that sense our approach resembles that of an event study in which one could capture the effect of an event (change in ESG score) on returns. We only expect an effect from ESG on returns if the information is relevant for the stock value. One interesting aspect this modelling is whether ESG can be interpreted as an additional risk factor. If a higher ESG score implies lower downside risk, this means that stock returns might be lower when ESG scores increase. This logic has not been yet to the best of our knowledge applied and tested in previous studies.

3 Data description

Monthly ESG scores of various firms from different countries and industries were obtained from Sustainanalytics which provides global research and data related to ESG and corporate governance. The data consist of ESG scores and scores of its three pillars ENV, SOC, GOV which are respectively related to Environmental, Social and Governance criteria.

The time frame of the collected data is from August 2009 to December 2017 and includes almost 900 stocks which are listed in five European countries: Sweden, Germany, France, United Kingdom and Netherlands. We thus provide European evidence regarding the sustainability effect on firms' risk and return.

Our motivation for the choice of stocks from these five countries is that they have relative stable economies which mitigates return fluctuations due to systemic risks such as political or economic turmoil during the period of the study. Thus, we assume that the five countries better capture return changes related to idiosyncratic movements of each firm performance.

The relevant stock prices were obtained from Thomson Reuters Datastream. The SMB and HML factors used in the the Fama and French three-factor model were collected from the Fama and French database (European three factors) apart from the case of Sweden and UK for which the factors were obtained respectively from the Swedish House of Finance (SHoF) and the Xfi Centre for Finance and Investment of the University of Exeter. In terms of the market factor the following indices were used: the HDAX index for the German market, the CACall for the French, the FTSE250 for the British, the SIXRX index for the Swedish and the AEXall for the Dutch. The risk-free rate is defined by the 10-year government yield of the respective country.

Table 1: Total number of stocks and observations per country

	Stocks (n)	Obs.(N)
Sweden (SWE)	81	4,131
Germany (GE)	164	10,877
France (FR)	144	10,387
United Kingdom (UK)	396	22,236
Netherlands (NED)	92	5,515
Total (EU)	877	53,146

3.1 Descriptive statistics

Table 2: Descriptive statistics: ESG rating and stock returns

		Mean	Median	SD^1	Min	Max	Observations
ΔESG	Overall	0.002	0	0.021	-0.363	0.513	$N = 53,088$
	Between			0.004	-0.041	0.085	$n = 869$
	Within			0.021	-0.341	0.510	$\bar{T} = 61.091$
ΔENV	Overall	0.002	0	0.033	-0.522	0.892	$N = 53,088$
	Between			0.007	-0.063	0.151	$n = 869$
	Within			0.033	-0.521	0.887	$\bar{T} = 61.091$
ΔGOV	Overall	0.001	0	0.024	-0.342	0.472	$N = 53,088$
	Between			0.005	-0.031	0.035	n 869
	Within			0.024	-0.310	0.460	$\bar{T} = 61.091$
ΔSOC	Overall	0.002	0	0.030	-0.476	1.059	$N = 53,088$
	Between			0.004	-0.022	0.065	$n = 869$
	Within			0.030	-0.481	1.048	$\bar{T} = 61.091$
R_i	Overall	0.009	0.0075	0.104	-0.100	9.441	$N = 48,949$
	Between			0.030	-0.328	0.400	$n = 788$
	Within			0.103	-0.974	9.441	$\bar{T} = 62.118$

Overall:

Note: This table displays the mean, the median, the standard deviation, as well as the minimum and maximum return values of the first difference of log ESG score and its individual pillars. Overall it refers to the full dataset, between to the cross-section, and within to the time-series dimension of each stock. N indicates the total number of observations, n the number of firms and $\bar{T}$ the average length of each time-series in months.

As the standard deviation (SD) in Table 2 shows, ESG scores' variability is very limited compared to the firms' stock returns. The overall, between and within volatility of firm's return rate are three to five times higher compared to aggregate ESG and to its individual components as well. Furthermore, the mean and median of all ESG scores are much closer to zero than these of the stock returns. The range of the ESG scores values is also smaller than that of the stock returns, additionally highlighting the presence of low ESG variability.

Table 3 reveals that the mean absolute value of each VaR_θ level decreases successively over the period. Until 2012, all mean values are higher than those following 2013, implying larger amounts of capital were needed to absorb of potential extreme price variations. This is possibly a result from the global financial crisis of 2007–2009, when there were many sizable fluctuations in stock returns captured by our VaR models. As the moving window utilized lasts approximately four years (1000 trading days), the datasets of 2011 and 2012 contain the majority of such considerable fluctuations from 2007, producing higher VaR estimates in many cases.

The standard deviation does not follow the same pattern as the mean. This different patterm of variation may be due to idiosyncratic shifts due to the large number of different companies included in each dataset. Indeed, the SD between panel which refers to the SD differentiation among firms is higher than the SD within panel in almost all the cases, driving the overall SD upwards. In addition, VaR estimates increase as the VaR level rises, as the probability of suffering losses over the VaR estimates decreases.

Table 3: Descriptive statistics for $VaR_{0.95}$

Total number of observations (T): 37,975

Total number of stocks (n): 555

T-bar ($\bar{T}$): 68.423

year	2009	2010	2011	2012	2013	2014	2015	2016	2017
Mean (in abs.)	0.033	0.030	0.035	0.031	0.027	0.026	0.029	0.030	0.024
SD overall	0.014	0.011	0.016	0.017	0.012	0.013	0.021	0.016	0.012
SD between panels	0.011	0.008	0.011	0.013	0.011	0.010	0.013	0.012	0.011
SD within panels	0.008	0.008	0.011	0.010	0.007	0.008	0.017	0.011	0.007
Min	-0.191	-0.106	-0.194	-0.354	-0.243	-0.356	-1.164	-0.328	-0.301
Max	0	0	0	-0.003	-0.005	-0.005	-0.006	0	0
N	1150	2990	3698	4382	5137	5313	5594	5693	4018
n	238	267	342	436	439	466	487	499	479
$\bar{T}$	4.832	11.199	10.813	10.051	11.702	11.401	11.487	11.409	8.388

Table 4: Descriptive statistics for $VaR_{0.99}$

Total number of observations (T): 46,091

Total number of stocks (n): 686

T-bar ($\bar{T}$): 67.188

year	2009	2010	2011	2012	2013	2014	2015	2016	2017
Mean (in abs.)	0.058	0.052	0.059	0.056	0.044	0.043	0.047	0.050	0.043
SD overall	0.033	0.024	0.030	0.151	0.019	0.020	0.035	0.028	0.024
SD between panels	0.028	0.018	0.022	0.178	0.016	0.016	0.025	0.021	0.021
SD within panels	0.017	0.017	0.021	0.017	0.011	0.012	0.028	0.020	0.013
Min	-0.415	-0.558	-0.587	-4.354	-0.344	-0.457	-1.906	-0.546	-0.421
Max	0	0	0	0	0	-0.007	0	0	0
N	1411	3588	4425	5347	6376	6507	6836	6815	4786
n	292	322	413	539	546	574	599	599	570
$\bar{T}$	4.832	11.143	10.714	9.920	11.678	11.336	11.412	11.377	8.396

Table 5: Descriptive statistics for $VaR_{0.995}$

Total number of observations (T): 46,542

Total number of stocks (n): 695

T-bar ($\bar{T}$): 66.967

year	2009	2010	2011	2012	2013	2014	2015	2016	2017
Mean (in abs.)	0.087	0.079	0.085	0.073	0.055	0.055	0.057	0.061	0.053
SD overall	0.087	0.088	0.124	0.175	0.025	0.028	0.045	0.036	0.032
SD between panels	0.084	0.080	0.122	0.199	0.022	0.025	0.029	0.028	0.033
SD within panels	0.025	0.030	0.038	0.045	0.012	0.015	0.036	0.025	0.016
Min	-1.055	-1.255	-2.210	-4.821	-0.448	-0.616	-2.578	-0.738	-0.780
Max	0	0	0	0	0	0	0	0	0
N	1401	3588	4441	5383	6460	6591	6899	6901	4878
n	290	324	414	547	552	582	605	608	576
$\bar{T}$	4.831	11.074	10.727	9.841	11.70	11.325	11.40	11.35	8.469

4 Results

4.1 Value-at-Risk and ESG score

Table 6 presents the results for the overall ESG score, while the estimates for the score components are displayed in Tables 9 to 11. Column 1 reports the results for Swedish companies, column 2 shows results for the entire sample, while the other columns display results for Germany, France, UK and the Netherlands, respectively. Instantaneous estimates for ESG effects are presented in the first row of all tables, followed by lags 1, 2 and 3.

It is important to note that the higher is the VaR of an investment, the riskier the investment is considered to be as VaR reveals the percentage loss expected from negative price fluctuations. Consequently, investing in stocks whose aforementioned ESG scores are high may lead to the accumulation of less capital for facing potential risks which could be deployed to other opportunities.

The value at risk estimates that are significantly different from zero reported in Table 6 are mainly negative. The economic interpretation is that firms' downside risk decreases with a positive change of ESG rating. The 95%-VaR point estimate for Swedish companies displayed in the upper panel, is -0.416, significant at the 10% level, indicating that a 1% higher ESG score is associated with 0.4% lower 95%-VaR.

This negative relation is statistically significant in many cases for at least one of the three lags. The effect of the second lag appears to be much stronger than that of the other variables having either the ESG or SOC score as independent variable, suggesting that there may be approximately a two-month delay for these scores to have an actual impact on market risk. This seems reasonable as changing corporate policies may need some time until they practically affect firms' financial performance. Similarly, the third lag of the ENV scores (Table 9) looks to be more important than the others, implying the relevant impact delay on VaR.

The GOV score has a positive coefficient when it is statistically significant (Table 11), meaning that increasing its value may increase firms' downside risk. This contradicts some of the existing literature supporting the hypothesis that better firm governance improves their risk profile. This positive association highlighted in the Table 11 may give signs of inefficiency for firms with a high governance score due to a possible lack of flexibility to respond and adapt to changing conditions, as VaR expresses the risk for cases of large price fluctuations.

In addition, the constant term is in all cases positive and highly significant, highlighting the conditional mean of VaR when exogenous control variables are included. Comparing the countries to the other, Swedish stocks' risk appears to be more sensitive to ESG score than stocks in other European countries considering the values and the statistical significance of the ESG coefficients. In terms of the ESG overall score, Swedish stocks' downside risk is sensitive in all lags. In contrast, Dutch stocks' risk is not affected by ESG scores' variation in almost all cases. Furthermore, the VaR of stocks listed in Germany appears to be more dependent on the second lag of the ESG scores than on other lags or on current ESG values.

Regarding, the sample of companies listed in the United Kingdom, the tables suggest that different VaR levels alter lag importance. The Environment score does not appear to have any effect on market risk of firms listed in France, and thus the negative impact of ESG score on VaR is most possibly driven by the Social score. Additionally, Table 11 implies that the Governance score effect on VaR is insignificant for firms in Sweden and apparently for those in the Netherlands.

4.2 Risk-adjusted return and ESG score

According to the results for the Eq. (15) model shown in Table 12, overall ESG scores have a very limited impact on companies' risk-adjusted return. Considering the pooled sample containing the stocks of all countries, these scores are not statistically significant. Nevertheless, considering each country individually, it can be seen that for some countries individual ESG pillars appear to have a significant effect on returns. For example, the ESG variable has a positive and statistically significant effect on risk-adjusted returns of companies listed in Sweden. In particular, a 1% increase in ESG score causes a 0.085% increase in stocks' risk adjusted return. ESG scores also seems to be more important for these firms than the HML factor of the established three-factor returns model. Comparing each ESG pillar for Sweden, it is evident that this impact tends to be driven by the ENV score (Table 13), suggesting signs of better returns' performance for stocks with high environmental scores compared to other stocks in this market.

In contrast, the same ENV score appears to negatively affect German firms' returns: a controversial result considering some of the existing literature about the positive effects of sustainability on firms' financial performance. Moreover, the results for the other three countries (France, United Kingdom and the Netherlands) suggest that stock returns' changes are irrelevant to ESG scores' changes as there is no evidence of a statistically significant relation from the latter to the former. Finally, with respect to the other factors, the market factor and the SMB factor appear to be the most important ones in this model.

5 Conclusions

The business sector is likely to be the key component of climate policy. A main challenge is to make the transition process to sustainable economic growth compatible with standard profit maximisation behaviour. This paper investigates whether corporate social responsible behaviour expressed by Environmental, Social and Governance (ESG) scores that are beneficial for a broad group of stakeholders are also advantageous for investors.

There is evidence that firms and their owners increasingly focus on sustainability issues. An illustrative example is that the number of companies considering corporate social responsibility continues to increase worldwide. Currently this is most prevalent in Europe, where more than 50% of professional managed funds are using socially responsible investment strategies according to statistics from Eurosif, an association for the promotion and advancement of sustainable and responsible investment across the European region.

The paper tests two hypotheses. The first is that ESG is inversely related to firms' downside risk and the second is that ESG is not related to risk-adjusted return. Downside risk is estimated by a Value at Risk (VaR) model and risk-adjusted return by the Fama–French three factor model.

For investors, reduced downside risk is beneficial as it reduces the likelihood that extreme negative returns occur. For the companies, reduced downside risk might be beneficial because it leads to a lower cost of capital. If a company's downside risk is reduced when ESG increases, not only equity investors, but also debt holders such as banks will require lower interests on the company's loans.

The capital asset pricing model (CAPM), commonly used to determine a theoretically required rate of return of an asset and for making decisions about adding assets to a well-diversified portfolio, predicts that lower risk, measured by stock price volatility should be associated with lower required return on the stock, and vice versa. Thus, successful ESG engagement will be considered by investors as a safeguard for the limited downside risk of their investment.

Using the daily historical returns of 887 stocks listed on five European stock markets over the period 2005 to 2017, we compute Value at Risk (VaR_θ) estimates for each stock from 2009 to 2017 by using a backtesting process. We set a moving window of 1000 trading days in which we employ autoregressive time-series techniques to obtain the necessary 1-day forecasts of stock returns and variance for computing the VaR for each stock.

The empirical findings of our econometric analysis reveal that firms with positive changes of ESG have lower financial downside risks as described by VaR. An important implication of reduced downside risk is that firms can lower their capital costs, not only on equity markets but also with respect to debt. For instance, a bank might be willing to give a firm a loan with lower interest rates if ESG scores of that firm are high and thus the risk of the firm, in terms of good environmental, social and governance practices, is low. In the equity market, a growing number of investment funds consider ESG factors in their investment strategies, which may lead to an increased focus on companies that are managing environmental and social issues effectively and have strong corporate governance practices. As our study shows, these firms tend to be lower-volatility and presumably higher-quality companies that fare better during downturns.

In contrast to numerous previous studies, the analysis reveals no relationship between change of ESG and a stock's risk-adjusted return. However, these results are in line with the theoretical prediction of the capital asset pricing model (CAPM), as extended to the Fama–French three factor model in our analysis.

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Appendix

I Tables

Table 6: ESG regression results for $VaR_{0.95}$ (fixed effects models)

	Dep. var.: $VaR_{0.95}$					
	SWE	EU	GE	FR	UK	NED
ESG	-0.416*	0.081	-0.102	0.478***	0.183	-0.425
	(0.215)	(0.108)	(0.139)	(0.163)	(0.242)	(0.308)
ESG(t-1)	-0.443**	0.025	0.344	0.200	-0.190	-0.011
	(0.198)	(0.079)	(0.220)	(0.161)	(0.118)	(0.189)
ESG(t-2)	-0.813***	-0.602***	-0.539**	-0.667***	-0.580***	-0.499
	(0.204)	(0.109)	(0.207)	(0.173)	(0.222)	(0.310)
ESG(t-3)	-0.014	-0.105	-0.100	-0.016	-0.248**	0.065
	(0.246)	(0.072)	(0.152)	(0.128)	(0.112)	(0.302)
Constant	0.035***	0.034***	0.033***	0.027***	0.034***	0.052***
	(0.001)	(0.000)	(0.000)	(0.001)	(0.000)	(0.001)
N	2,874	35,625	7,614	7,436	14,293	3,396
R-squared	0.003	0.001	0.002	0.003	0.001	0.001
n stocks	55	541	114	94	227	54

Table 7: ESG regression results for $VaR_{0.99}$ (fixed effects models)

	Dep. var.: $VaR_{0.99}$					
	SWE	EU	GE	FR	UK	NED
ESG	-0.377*	0.021	-0.017	0.201	0.105	-0.359
	(0.229)	(0.091)	(0.131)	(0.167)	(0.208)	(0.231)
ESG(t-1)	-0.328*	0.014	0.334*	0.177	-0.196*	0.046
	(0.190)	(0.068)	(0.197)	(0.145)	(0.109)	(0.141)
ESG(t-2)	-0.539***	-0.490***	-0.490***	-0.551***	-0.540***	-0.338
	(0.193)	(0.094)	(0.187)	(0.173)	(0.199)	(0.248)
ESG(t-3)	0.039	-0.077	-0.062	0.028	-0.170*	-0.020
	(0.247)	(0.061)	(0.143)	(0.112)	(0.091)	(0.217)
Constant	0.029***	0.032***	0.033***	0.024***	0.032***	0.048***
	(0.001)	(0.000)	(0.000)	(0.001)	(0.000)	(0.001)
N	3,229	43,168	9,032	8,861	17,638	4,452
R-squared	0.002	0.001	0.001	0.002	0.001	0.000
n stocks	64	666	138	114	285	70

Table 8: ESG regression results for $VaR_{0.995}$ (fixed effects models)

	Dep. var.: $VaR_{0.995}$					
	SWE	EU	GE	FR	UK	NED
ESG	-0.319*	-0.172	-0.178	0.094	0.194	-1.774
	(0.160)	(0.228)	(0.132)	(0.226)	(0.201)	(1.785)
ESG(t-1)	-0.313**	-0.194**	0.169	-0.204	-0.198*	-0.638
	(0.150)	(0.095)	(0.154)	(0.274)	(0.114)	(0.422)
ESG(t-2)	-0.456**	-0.438***	-0.374**	-0.751***	-0.318	-0.428
	(0.195)	(0.106)	(0.187)	(0.233)	(0.197)	(0.353)
ESG(t-3)	-0.067	-0.157**	-0.177	-0.126	-0.142	-0.222
	(0.196)	(0.075)	(0.139)	(0.196)	(0.086)	(0.330)
Constant	0.021***	0.040***	0.030***	0.061***	0.026***	0.093***
	(0.001)	(0.001)	(0.000)	(0.002)	(0.000)	(0.005)
N	3,331	43,535	8,944	8,794	18,128	4,382
R-squared	0.002	0.000	0.000	0.000	0.001	0.000
n stocks	66	676	137	112	296	70

Table 9: ENV score regression results for $VaR_{0.99}$ (fixed effects models)

	Dep. var.: $VaR_{0.99}$					
	SWE	EU	GE	FR	UK	NED
ENV	-0.258*	-0.022	-0.077	0.091	0.032	-0.186
	(0.130)	(0.063)	(0.097)	(0.105)	(0.103)	(0.383)
ENV(t-1)	-0.315***	-0.071	0.186	-0.065	-0.147**	-0.113
	(0.118)	(0.046)	(0.155)	(0.076)	(0.065)	(0.125)
ENV(t-2)	-0.134	-0.099*	-0.323***	-0.164	0.072	-0.148
	(0.114)	(0.060)	(0.106)	(0.136)	(0.112)	(0.154)
ENV(t-3)	-0.064	-0.167***	-0.112	-0.083	-0.252***	-0.223*
	(0.128)	(0.037)	(0.097)	(0.073)	(0.059)	(0.119)
Constant	0.029***	0.032***	0.033***	0.025***	0.032***	0.049***
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)
N	3,229	43,168	9,032	8,861	17,638	4,452
R-squared	0.002	0.000	0.001	0.000	0.001	0.000
n stocks	64	666	138	114	285	70

Table 10: SOC score regression results for $VaR_{0.99}$ (fixed effects models)

	Dep. var.: $VaR_{0.99}$					
	SWE	EU	GE	FR	UK	NED
SOC	-0.102	0.008	0.089	0.044	0.011	-0.215
	(0.147)	(0.062)	(0.104)	(0.100)	(0.127)	(0.197)
SOC(t-1)	-0.134	0.041	0.146	0.175^{*}	-0.033	0.053
	(0.122)	(0.053)	(0.177)	(0.100)	(0.082)	(0.096)
SOC(t-2)	-0.423^{**}	-0.484^{***}	-0.352^{*}	-0.388^{***}	-0.757^{***}	-0.225
	(0.166)	(0.076)	(0.181)	(0.108)	(0.151)	(0.169)
SOC(t-3)	0.214	0.077^{*}	0.114	0.100	0.061	0.083
	(0.206)	(0.044)	(0.106)	(0.078)	(0.060)	(0.141)
Constant	0.028^{***}	0.032^{***}	0.033^{***}	0.024^{***}	0.032^{***}	0.047^{***}
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)
N	3,229	43,168	9,032	8,861	17,638	4,452
R-squared	0.002	0.002	0.001	0.002	0.004	0.000
n stocks	64	666	138	114	285	70

Table 11: GOV score regression results for $VaR_{0.99}$ (fixed effects models)

	Dep. var.: $VaR_{0.99}$					
	SWE	EU	GE	FR	UK	NED
GOV	0.177	0.207^{***}	0.068	0.341^{***}	0.196^{***}	0.167
	(0.190)	(0.052)	(0.105)	(0.089)	(0.084)	(0.257)
GOV(t-1)	0.139	0.170^{***}	0.318^{*}	0.203^{*}	0.067	0.269
	(0.161)	(0.061)	(0.167)	(0.111)	(0.101)	(0.196)
GOV(t-2)	-0.196	0.046	0.004	-0.015	0.234	-0.209
	(0.179)	(0.074)	(0.146)	(0.130)	(0.146)	(0.222)
GOV(t-3)	0.027	0.003	-0.134	0.092	-0.025	0.160
	(0.180)	(0.049)	(0.123)	(0.087)	(0.076)	(0.175)
Constant	0.027^{***}	0.031^{***}	0.032^{***}	0.023^{***}	0.031^{***}	0.046^{***}
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)
N	3,229	43,168	9,032	8,861	17,638	4,452
R-squared	0.001	0.000	0.000	0.001	0.000	0.000
n stocks	64	666	138	114	285	70

Table 12: FF three factor model and ESG scores regression results for excess returns

Dep. var.: $r_{i,t} - r_{f,t}$	SWE	EU	GE	FR	UK	NED
Rm-Rf	1.013***	0.919***	0.795***	0.948***	0.961***	0.938***
	(0.045)	(0.016)	(0.026)	(0.036)	(0.032)	(0.052)
SMB	-0.097***	0.103***	0.570***	0.271***	0.059***	0.216***
	(0.027)	(0.017)	(0.086)	(0.044)	(0.019)	(0.060)
HML	0.035	0.049***	0.136**	0.008	0.048*	0.131***
	(0.035)	(0.018)	(0.055)	(0.031)	(0.029)	(0.047)
ΔESG	0.085*	-0.050	-0.040	-0.016	-0.100	-0.021
	(0.047)	(0.084)	(0.045)	(0.032)	(0.230)	(0.059)
Constant	-0.002***	-0.001***	-0.003***	0.001*	0.100	-0.002***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.230)	(0.001)
N	4,131	53,146	10,877	10,387	22,236	5,515
R-squared	0.289	0.145	0.171	0.268	0.089	0.164
n	81	877	164	144	396	92

Table 13: FF three factor model and ENV scores regression results for excess returns

Dep. var.: $r_{i,t} - r_{f,t}$	SWE	EU	GE	FR	UK	NED
Rm-Rf	1.013***	0.919***	0.797***	0.947***	0.961***	0.939***
	(0.045)	(0.016)	(0.027)	(0.036)	(0.031)	(0.052)
MB	-0.095***	0.102***	0.564***	0.270***	0.057***	0.219***
	(0.027)	(0.017)	(0.085)	(0.044)	(0.019)	(0.060)
HML	0.036	0.049***	0.134**	0.007	0.049*	0.132***
	(0.035)	(0.018)	(0.055)	(0.031)	(0.029)	(0.047)
ΔENV	0.058*	-0.044	-0.064**	-0.019	-0.071	0.024
	(0.029)	(0.050)	(0.025)	(0.019)	(0.121)	(0.038)
Constant	-0.002***	-0.001***	-0.003***	0.001*	0.071	-0.002***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.121)	(0.001)
N	4,131	53,146	10,877	10,387	22,236	5,515
R-squared	0.289	0.145	0.171	0.268	0.089	0.164
n	81	877	164	144	396	92

Table 14: FF three factor model and SOC scores regression results for excess returns

Dep. var.: $r_{i,t} - r_{f,t}$	SWE	EU	GE	FR	UK	NED
Rm-Rf	1.014***	0.919***	0.795***	0.948***	0.963***	0.937***
	(0.045)	(0.016)	(0.027)	(0.036)	(0.031)	(0.052)
SMB	-0.097***	0.104***	0.571***	0.271***	0.059***	0.219***
	(0.027)	(0.017)	(0.085)	(0.044)	(0.019)	(0.060)
HML	0.034	0.048***	0.135**	0.007	0.046*	0.130***

Continued on next page

Table 14 – *Continued from previous page*

	SWE	EU	GE	FR	UK	NED
	(0.035)	(0.017)	(0.055)	(0.031)	(0.026)	(0.047)
ΔSOC	0.025	-0.038	-0.038	-0.015	-0.060	-0.044
	(0.031)	(0.057)	(0.025)	(0.023)	(0.152)	(0.036)
Constant	-0.002***	-0.001***	-0.003***	0.001*	0.060	-0.002***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.152)	(0.001)
N	4,131	53,146	10,877	10,387	22,236	5,515
R-squared	0.289	0.145	0.171	0.268	0.089	0.164
n	81	877	164	144	396	92

Table 15: FF three factor model and GOV scores regression results for excess returns

Dep. var.: $r_{i,t} - r_{f,t}$						
	SWE	EU	GE	FR	UK	NED
Rm-Rf	1.015***	0.920***	0.794***	0.949***	0.965***	0.939***
	(0.045)	(0.016)	(0.027)	(0.036)	(0.029)	(0.052)
SMB	-0.100***	0.103***	0.572***	0.267***	0.059***	0.216***
	(0.028)	(0.017)	(0.085)	(0.044)	(0.019)	(0.060)
HML	0.035	0.049***	0.138**	0.007	0.045*	0.132***
	(0.035)	(0.017)	(0.055)	(0.031)	(0.026)	(0.047)
ΔGOV	0.069	0.007	0.040	0.014	-0.028	0.005
	(0.042)	(0.027)	(0.050)	(0.023)	(0.063)	(0.048)
Constant	-0.002***	-0.001***	-0.003***	0.001*	0.028	-0.002***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.063)	(0.000)
N	4,131	53,146	10,877	10,387	22,236	5,515
R-squared	0.289	0.145	0.171	0.268	0.089	0.164
n	81	877	164	144	396	92

II Christoffersen test

The Christoffersen test consists of a sequence of two partial tests: The unconditional coverage and the test of independence. Proceeding to these tests, we started calculating the number of violations occurred in each method. This expresses the number of times when the model failed to predict the return loss occurred. For this purpose, the so-called "hit function" was used. The sequence $\{Hit_{t+1}\}_{t=1}^{T}$ is *iid* Bernoulli(p).

$$\{Hit_{t+1}\}_{t=1}^{T} = \begin{cases} 1 & \text{if} r_t < -\overline{VaR_{\theta,t}} \quad \text{(violation occurs)} \\ 0 & \text{if} r_t > -\overline{VaR_{\theta,t}} \quad \text{(no violation occurs)} \end{cases} \tag{16}$$

The number of exceedances can be expressed as:

$$x = \sum\nolimits_{t=1}^{T} Hit_t \tag{17}$$

1) According to the unconditional coverage property, the probability of facing a loss not exceeding $VaR(\theta)$ needs to be approximately $(1-\theta)$. For instance, if a confidence level of $\theta = 99\%$ is used, the null hypothesis is that the frequency of tail losses is equal to $p = 1 - \theta = 1\%$. Assuming that the model is accurate, the observed failure rate x/T should act as an unbiased measure of p, and thus converge to 1% as sample size increases. If the forecasted VaR exceeds too many times the actual return losses, then there is evidence that the model too conservative. In such a case, the model tend to overestimate the VaR which usually leads to the conclusion that more capital is needed for covering such potential losses. The null hypothesis of the former test is $H_0 : E[Hit_t] = p$, with the LR statistic illustrated as follow:

$$LRunc = -2log\left(\frac{L(p|Hit_t, t = 1, ...T)}{L(\bar{\pi}|Hit_t, t = 1, ...T)}\right) \tag{18}$$

where p is the confidence level T the number of VaR estimations in this case is: $L(p|Hit_t, t = 1, ...T)$ $(1-p)^{n_0} p^{n_1}$ and $L(\bar{\pi}|Hit_t, t = 1, ...T) = (1-\bar{\pi})^{n_0} \bar{\pi}^{n_1}$

and $\bar{\pi} = n_1/(n_0 + n_1)$ is the MLE of Π, the true exceedance probability. Clearly, n_1 is the number of exceedances and $n_0 = T - n_1$.. Under the null hypothesis, LR_{unc} is asymptotically distributed as a $\chi^2(1)$.

2) With respect to the test of independence, the independence of the Hit_t is tested. Under the alternative hypothesis, the Hit_t is a two-state Markov chain, with transition probability matrix:

$$\pi_1 = \begin{bmatrix} 1-\pi_{01} & \pi_{01} \\ 1-\pi_{11} & \pi_{11} \end{bmatrix} \quad \text{where} \quad \pi_{ij} = P_r[Hit_t = j | Hit_{t-1} = i] \tag{19}$$

Therefore, the likelihood function for this process is as follow:

$$L(\Pi_1 | Hit_t, t = 1, ...T) = (1-\pi_{01})^{n_{00}} (\pi_{01})^{n_{01}} (1-\pi_{11})^{n_{10}} (\pi)_{11})^{n_{11}} \tag{20}$$

where n_{ij} is the number of observations with value i followed by value j. The probability π_{ij} is estimated by $\bar{\pi}_{ij} = n_{ij}/(n_{io} + n_{i1})$.

Under the null of independence, the transition probability matrix reduces to:

$$\Pi_2 = \begin{bmatrix} 1-\pi_2 & \pi_2 \\ 1-\pi_2 & \pi_2 \end{bmatrix} \tag{21}$$

with likelihood function:

$$L(\Pi_1 | Hit_t, t = 1, ...T) = (1-\pi_2)^{n_{00}+n_{10}} (\pi_2)^{n_{01}+n_{11}} \tag{22}$$

The probability π_2 is estimated by $\bar{\pi}_2 = (n_{01} + n_{11})/T$. Then, the LR statistic is:

$$LRind = -2log \left(\frac{L(\widehat{\Pi_2} | Hit_t, t = 1, ...T)}{L(\widehat{\Pi_1} | Hit_t, t = 1, ...T)} \right) \tag{23}$$

Under the null hypothesis, LR_{ind} is asymptotically distributed as a $\chi^2(1)$. Finally, the joint test of coverage and independence, which corresponds to the test of conditional coverage, is given by the test statistic LR_{cc}. The LR_{cc} is asymptotically distributed as a $\chi^2(2)$ under the null hypothesis.

$$LR_{cc} = LR_{unc} + LR_{ind} = -2log \left(\frac{L(p) | Hit_t, t = 1, ...T)}{L(\widehat{\Pi_1} | Hit_t, t = 1, ...T)} \right) \tag{24}$$