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Reconsidering the returns to entrepreneurship: Applying a modified version of Lazear's occupational choice model

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Applying a modified version of Lazear's occupational choice model

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Abstract: The paper analyzes a modified version of Lazear's (2004, 2005) model for occupational choice that includes a utility adjustment factor reflecting non-pecuniary net benefits associated with entrepreneurship. The model is used to derive analytical expressions for the relative income returns to entrepreneurship defined in two ways: in relation to the incomes of observationally similar wage employed and in relation to the counterfactual incomes of entrepreneurs as wage employed. The analysis shows that the upper bound on the income returns defined in counterfactual terms increases with the market value of entrepreneurial talent and that the lower bound decreases with increasing non-pecuniary benefits. If the entrepreneurial incomes are instead related to the incomes of observationally similar wage employed, and if the skill profiles in the population are assumed to follow a Fréchet distribution, then the upper bound on the relative expected returns will decrease with increasing non-pecuniary benefits. We also show that entrepreneurs on average will earn less than observationally similar wage employed even if there are no net benefits and that the associated self-selection bias will increase the higher the fraction of entrepreneurs. Individual-based data from the Swedish employment register is used to calibrate the parameters of the modified Lazear model and to compute the income returns to entrepreneurship. The model-based income distributions for entrepreneurs and wage employed are consistent with the observed distributions.

Keywords: entrepreneurship; occupational choice; self-employment; self-selection bias; skill distribution; Fréchet distribution; entrepreneurship puzzle; returns to entrepreneurship
JEL Codes: M13; J24; J3; L26

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1. Introduction

This paper analyzes the returns to entrepreneurship by applying a utility-based version of Lazear's (2002, 2004, 2005) model for occupational choice. We include a multiplicative factor that adjusts entrepreneurial incomes for non-pecuniary benefits, investigate two measures of the returns to entrepreneurship and make explicit assumptions for the distribution of skills in a population.

Lazear assumes that individuals have two basic skills and that their choice between entrepreneurship and wage employment is determined by the resulting income. Individuals that choose wage employment will be paid according to the strength of their strongest skill, and the payment for those that choose entrepreneurship will equal the strength of their weakest skill times a positive parameter reflecting the market value of entrepreneurial talent.[1] The income maximization implies that individuals having balanced skills are more likely than others to become entrepreneurs.

Different aspects of Lazear's model have been tested in a large number of empirical studies and most of them seem to support the implication that entrepreneurs have more balanced skills than wage employees; see Lazear (2004, 2005), Wagner (2003, 2006), Cho and Orazem (2014), Bublitz and Noseleit (2014) and Aldén et al. (2017), for instance. The model has also been criticized. A long-standing issue in the literature relates to the observation that entrepreneurs tend to earn less than comparable wage employees; see Hamilton (2000) and Åstebro (2012). Hamilton (2000) suggests that non-pecuniary benefits of being one's own boss might compensate for the observed earning differences. Hartog et al. (2010) claim that non-pecuniary benefits seem to be necessary to solve "the returns-to-entrepreneurship puzzle" and Benz (2009) states that "…entrepreneurship cannot possibly be understood as a quest for profit alone" (page 24).

The criticism seems to exclude an occupational choice model based on income maximization alone. However, it should be noted that Lazear (2002) shows that whether his model will result in higher or lower average income for entrepreneurs than for wage employed depends upon the assumption made about the distribution of skill profiles in the population. Using numerical methods, he shows that a normal distribution will always imply a higher average income for entrepreneurs than for wage employed, whereas a gamma distribution, under some conditions, will reverse that relationship.

Another common explanation for the observed earning differential is related to the difficulty in measuring entrepreneurial earnings and to the definition of an entrepreneur; see Parker (2009) for an overview. Lechman (2015) reports that the contribution of different working conditions to the earning gap is marginal and concludes that income underreporting might play the main role. According to Berglann et al. (2011) and Åstebro and Chen (2014), entrepreneurs might in fact earn more than comparable wage employees provided that their incomes are measured accurately. Levine and Rubenstein (2016) differentiate between incorporated and unincorporated self-employed and show that the incorporated self-employed earn more and the unincorporated less than comparable wage employed.

In addition to the measurement problem, unobserved heterogeneity makes it difficult to estimate what entrepreneurs would have earned if they instead had chosen to become wage employed. Hyytinen et al. (2013) correct for the heterogeneity caused by genetic factors by using a database for identical twins that includes information about earnings as well as working conditions. They report negative earning returns but compensating positive non-pecuniary benefits for entrepreneurs.

[1] Risk considerations by the individuals are ignored in the Lazear model.

One aim of the present paper is to derive analytical expressions showing how the relative income returns to entrepreneurship depend upon the assumption made about non-pecuniary net benefits. The relative returns are defined in two ways: in relation to the counterfactual incomes the entrepreneurs would have earned as wage employed and in relation to the incomes of observationally similar wage employed. A second purpose is to clarify the self-selection bias characterizing the latter definition, and a third one to derive the theoretical implications of the assumption that the skills are Fréchet-distributed. Additionally, we will use Swedish individual level employment data to show some empirical implications of our model approach.

We will assume that the occupational choice is utility driven, and that the utility of being an entrepreneur equals the income, as in Lazear's model, but multiplied by a factor reflecting the extent of non-pecuniary net benefits. This modification is mathematically equivalent to the one suggested by Benz (2009) to handle non-monetary benefits caused by a "taste for variety". Additionally, we need to specify an appropriate statistical skill distribution. Earlier empirical studies of the income differences between entrepreneurs and wage employed and our own income distribution data provide some hints for the specification. First, the skill distribution should allow, but not require, lower average earnings of entrepreneurs than of observationally similar wage employed. It follows that neither a normal nor a uniform skill distribution would be appropriate since both imply a reversed income ratio as shown by Lazear (2002) and Åstebro and Thompson (2011), respectively. Furthermore, the specification should be compatible with commonly observed differences between the income distribution of entrepreneurs and wage employed. As shown by Hamilton (2000) and Parker (2009), for instance, the income distribution of entrepreneurs typically has a larger variance and a larger positive skew. Another consistent feature is an overrepresentation of the entrepreneurs in the lower and upper parts of the overall income distribution. By and large, similar differences hold for our Swedish income distribution data.

The Fréchet distribution, which is positively skewed with a fat right-hand tail, has the potential of fulfilling the stated conditions.[2] It is also attractive because its properties make it possible to relate the incomes of entrepreneurs and wage employed analytically to their counterfactual incomes or in other words to avoid the self-selection bias. Furthermore, the parameters of the Fréchet distribution are easy to calibrate against commonly available labor market data.

The observed income ratios between entrepreneurs and wage employed in our set are consistent with the assumption of Fréchet-distributed skills and it is also reasonably well supported by the similarity between the observed and the theoretically derived income distributions. The calibrated parameters indicate that negative aspects of entrepreneurship, such as risks, seem to be more common than positive non-pecuniary benefits among the population groups we study: electrical engineers (with a Master of Science degree), architects, and non-performing artists. We also show that the entrepreneurial supply curves for these groups differ considerably.

The paper contributes to earlier studies in several ways. It shows that the upper bound for the relative income returns to entrepreneurship – defined in terms of the counterfactual incomes – increases with the market value of entrepreneurial talent, and that the lower bound decreases with increasing non-pecuniary net benefits. The value of this measure for Fréchet-distributed skills is in fact close to the average of these two bounds. If the relative income returns to entrepreneurship instead are defined in relation to the incomes of observationally similar

[2] The Fréchet distribution is one of three univariate extreme value distributions that also include the Gumbel and Weibull distributions, see Kotz and Nadarajah (2000).

wage employed, and with the same skill distribution assumption, then the upper bound decreases with increasing non-pecuniary net benefits. Comparing our two measures of the relative income returns to entrepreneurship, we furthermore show that the self-selection bias inherent in empirical income returns studies comparing observationally similar entrepreneurs and wage employed is likely to be larger the higher the observed relative frequency of entrepreneurs.

The paper is organized as follows. Section 2 provides a description of the model framework and its theoretical implications. Section 3 presents our data and Section 4 the calculated values of the income returns, the utility adjustment factor and the other model parameters. Section 4 also includes a comparison between observed and theoretically derived income distributions. Concluding remarks follow in Section 5. Mathematical proofs and the applied procedure for parameter calibration are in Appendix A and additional data in Appendix B.

2. The model framework

Following Lazear (2005), we consider a population of individuals who choose between becoming entrepreneurs or wage employees. Each individual has two basic skills with strengths that vary in the population according to the positive random variables X_1 and X_2. The income as an entrepreneur is determined by the individual's weakest skill times a positive parameter λ representing the *market value of entrepreneurial talent*. For a randomly selected individual, the entrepreneurial income is thus $\lambda \cdot \min\{X_1, X_2\}$. As wage employed, on the other hand, the income will correspond to the strength of the individual's strongest skill, that is, $\max\{X_1, X_2\}$.

As discussed in the introduction, non-pecuniary benefits may explain why some individuals become entrepreneurs although their incomes as wage employed would be higher. Moreover, entrepreneurs' earnings, which are notoriously difficult to measure, are sometimes claimed to be higher than reported; see Åsterbro (2012), for instance. On the other hand, the risks associated with entrepreneurship might explain why some individuals choose to become wage employed although they could expect higher incomes as entrepreneurs. To allow for non-pecuniary benefits or dis-benefits as well as for possible underreporting of incomes, we make a simple adjustment of Lasear's occupational choice model, assuming that the utility of becoming a wage employed is

$$U_W = \max\{X_1, X_2\},$$

and the utility of becoming an entrepreneur is

$$U_E = s \cdot \lambda \cdot \min\{X_1, X_2\},$$

where the positive parameter s is a *utility adjustment factor* that represents how the individuals adjust the income (upwards $s > 1$, downwards $0 < s < 1$ or not at all $s = 1$) because of other factors influencing their utility.[3] The reason for assuming a multiplicative rather than, say, an additive adjustment is that it seems more likely that non-pecuniary benefits or dis-benefits will change the utility in proportion to the reported income rather than by an additive term. For example, it seems reasonable to assume that the benefits from being one's own boss

[3] A natural extension of this idea would be to assume a stochastic utility adjustment factor S, since there is realistically a variation in the utility adjustment among the individuals. This is beyond the scope of the present paper, however.

or from a "taste for variety" will increase with the income as is argued by Benz (2009).[4] In the sequel we will refer to s as a utility adjustment factor for non-pecuniary net benefits irrespective of whether the adjustment is related to benefits $(s > 1)$ or dis-benefits caused by, for instance, risks $(0 < s < 1)$. With this slight generalization of Lazear's model, an individual chooses entrepreneurship if and only if it gives a higher utility:

$$U_E > U_W .$$

We note that this could only happen if $s \cdot \lambda > 1$, which is therefore assumed. The probability that a randomly drawn individual chooses entrepreneurship is then

$$\begin{aligned} p_E &= \Pr[U_E > U_W] = \Pr[s \cdot \lambda \cdot \min\{X_1, X_2\} > \max\{X_1, X_2\}] \\ &= \Pr[s \cdot \lambda \cdot X_1 > X_2 > X_1] + \Pr[s \cdot \lambda \cdot X_2 > X_1 > X_2]. \end{aligned}$$

We note that the probability p_E is increasing in both s and λ.

In the following we let Y_E and Y_W be the *reported* income earned by (self-selected) entrepreneurs and (self-selected) wage employed, respectively.[5] Formally,

$$Y_E = \lambda \cdot [\min\{X_1, X_2\} | s \cdot \lambda \cdot \min\{X_1, X_2\} > \max\{X_1, X_2\}]$$

and

$$Y_W = [\max\{X_1, X_2\} | s \cdot \lambda \cdot \min\{X_1, X_2\} \le \max\{X_1, X_2\}].$$

Before introducing a specific skill distribution, we first only assume that the two random skills X_1 and X_2 are independent and identically distributed with strictly positive densities on the positive axes and with finite expected values.

The logic of the occupational choice model with these assumptions is illustrated in Figure 1. The probability of becoming an entrepreneur p_E is equal to the probability that (X_1, X_2) will fall between the two rays $X_2 = s \cdot \lambda \cdot X_1$ and $X_2 = X_1 / (s \cdot \lambda)$. Clearly, p_E goes from zero to one as $s \cdot \lambda$ goes from one to infinity, since then the rays will shift from coinciding with the diagonal to coinciding with the axes.

By definition, each individual chooses occupational status to maximize his utility. Does this mean that he also maximizes his income? If there are no non-pecuniary net benefits ($s = 1$), this is obviously true. When this is not the case $(s \ne 1)$, the situation is more intricate. To analyze this we first define the counterfactual income of an entrepreneur as the income he would have earned, if he instead had chosen to become wage employed

$$Z_E = [\max\{X_1, X_2\} | s \cdot \lambda \cdot \min\{X_1, X_2\} > \max\{X_1, X_2\}]$$

and the corresponding counterfactual income of a wage empoyed as

[4] Benz (2009) assumes that $U_E = (\eta + \lambda) \cdot \min\{X_1, X_2\}$, where the deterministic factor $\eta > 1 - \lambda$ accounts for non-monetary benefits. Our approach is mathematically equivalent to his, since $\eta + \lambda = s \cdot \lambda$ for $s = \eta / \lambda + 1$.

[5] It follows that income underreporting will influence the utility adjustment factor s upwards.

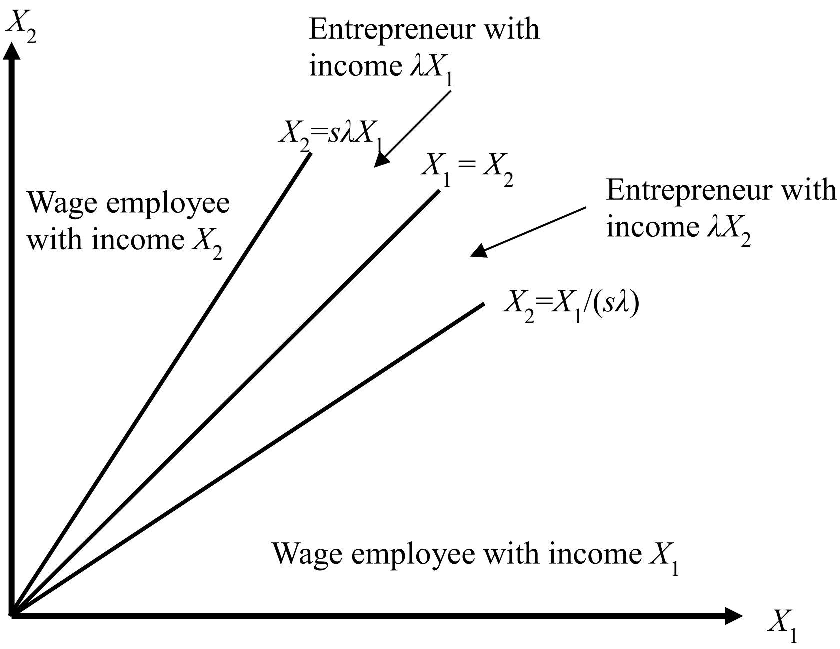

Figure 1. Self-selection into entrepreneurship or wage employment by individuals endowed with different skills X_1 and X_2 and with a utility adjustment factor s and a market value of entrepreneurship λ such that $s \cdot \lambda > 1$. Adapted from Lazear (2005)

$$\mathrm{Z}_W = \lambda \cdot [\min\{X_1, X_2\} \,|\, s \cdot \lambda \cdot \min\{X_1, X_2\} \leq \max\{X_1, X_2\}] .$$

Consider now an individual's income returns to entrepreneurship and to wage employment in relation to the counterfactual incomes, that is Y_E / Z_E and Y_W / Z_W, respectively. It should be noted that by construction these relative income returns will not be biased by self-selection.

Moreover, intuitive upper and lower bounds on them are easily derived. For an entrepreneur with perfectly balanced skills, that is, for whom $X_1 = X_2$, the relative income returns are $Y_E / \mathrm{Z}_E = \lambda$, since $\min\{X_1, X_2\} = \max\{X_1, X_2\}$. For an entrepreneur who is indifferent between being an entrepreneur or a wage employed, that is, for whom $s \cdot \lambda \cdot \min\{X_1, X_2\} = \max\{X_1, X_2\}$, the relative income returns are $Y_E / \mathrm{Z}_E = \lambda \cdot \min\{X_1, X_2\} / \max\{X_1, X_2\} = 1/s$. Hence

$$1/s < Y_E / \mathrm{Z}_E \leq \lambda .$$

Similarly, for an employee who is indifferent between being wage employed or an entrepreneur, that is, for whom $s \cdot \lambda \cdot \min\{X_1, X_2\} = \max\{X_1, X_2\}$, the relative income returns are $Y_W / \mathrm{Z}_W = \max\{X_1, X_2\} / \lambda \cdot \min\{X_1, X_2\} = s$. Hence

$$Y_W / \mathrm{Z}_W \geq s .$$

Based on these concepts of relative individual income returns we define the *relative expected income returns to entrepreneurship* as

$$\mu_E = \mathrm{E}[Y_E] / \mathrm{E}[\mathrm{Z}_E] ,$$

and the *relative expected income returns to wage employment* as

$$\mu_W = \mathrm{E}[Y_W]/\mathrm{E}[Z_W].$$ [6]

Obviously the inequalities for the relative individual income returns also hold for the relative expected income returns. In fact, the non-strict inequalities are strengthened to strict ones for the relative expected income returns, since the skill distributions have been assumed to have strictly positive densities. We have therefore established the following proposition:

Proposition 1. Consider our modified Lazear model with $s \cdot \lambda > 1$ and with random skill profiles $X_1 > 0$ and $X_2 > 0$ that are statistically independent and identically distributed with strictly positive densities on the positive axes and with finite expected values. The relative expected income returns to entrepreneurship μ_E then satisfy

$$1/s < \mu_E < \lambda,$$

and the relative expected income returns to wage employment satisfy

$$\mu_W > s.$$

It should noted that the inequalities hold for any continuous skill distribution as specified in the proposition. As we will see in the empirical application in Section 4, μ_E will be close to the average of its bounderies, that is, $\mu_E \approx (\lambda + 1/s)/2$.

If the utility adjustment factor $s \leq 1$, the first inequality guarantees that entrepreneurs on average will earn more than they would do as wage employed. If, on the other hand, $s > 1$, then then they can possibly earn less on average than they would do as wage employed. In either case their relative expected income returns will be smaller than the market value of entrepreneurship talent, that is, $\mu_E < \lambda$. The second inequality says that wage employed on average will earn more than they would do as entrepreneurs, if the utility adjusment factor $s \geq 1$. They could, on the other hand, on average earn less than they would do as entrepreneurs if $s < 1$ and hence μ_W might then be less than one. If $s = 1$, both entrepreneurs and wage employed will on average earn more than they would do by making the opposite occupational choice.

The possibility of self-selection bias is a recurrent problem in regression studies of earning differences between observationally similar entrepreneurs and wage employed and thus also for our second measure of income returns $\mathrm{E}[Y_E]/\mathrm{E}[Y_W]$. However, our first measure of income returns μ_E excludes self-selection by construction. It follows that the ratio between these two measures, that is $b = \mathrm{E}[Z_E]/\mathrm{E}[Y_W]$, provides information about the self-selection bias.

How does this measure of selection bias depend on the model parameters? That is difficult to say in the general case. To be able to analyze this question further, we need to make some assumption about the skill distributions. As already motivated, the Fréchet distribution seems to be a promising candidateto apply. We first derive closed-form expressions for the occupational choice probabilities given Fréchet-distributed skills.

[6] These relative expected income returns were introduced in Hårsman et al. (2018) for the situation when there are no non-pecuniary net benefits $(s = 1)$. Alternative definitions could be expected relative income returns $\tilde{\mu}_E = \mathrm{E}[Y_E/Z_E]$ and $\tilde{\mu}_W = \mathrm{E}[Y_W/Z_W]$. For Fréchet-distributed skills, the differences are small. In addition, the proposed definitions μ_E and μ_W are easier to apply mathematically.

Proposition 2. Consider our modified Lazear model with $s \cdot \lambda > 1$ and with random skills X_1 and X_2 that are statistically independent and identically *Fréchet* (β, v) -distributed with $\beta > 1$ and $v > 0$.[7] The probability of entrepreneurship and wage employment are then, respectively,

$$p_E = \frac{(s\lambda)^\beta - 1}{(s\lambda)^\beta + 1},$$

and

$$p_W = 1 - p_E = \frac{2}{(s\lambda)^\beta + 1}.$$

Proof. See Appendix A2.

As expected, the supply of entrepreneurs p_E increases at a decreasing rate with increasing non-pecuniary net benefits s and increasing market value of entrepreneurial talent λ. The same holds for β. The latter conclusion is intuitively clear, since the larger the value of β, the smaller is the variance of the skill distributions (see Appendix A1), which in turn will concentrate the probability mass around a small area on the diagonal in Figure 1.

With the same distributional assumptions, we can also derive closed-form expressions for the expected incomes of entrepreneurs and wage employed.

Proposition 3. Consider our modified Lazear model with the same assumptions as in Proposition 2. Then the expected incomes of entrepreneurs and wage employed are, respectively,

$$\mathrm{E}[Y_E] = \frac{1}{s} \cdot \frac{2(s\lambda)^\beta \cdot (1 + (s\lambda)^\beta)^{1/\beta} - 2^{1/\beta} s\lambda \cdot (1 + (s\lambda)^\beta)}{(s\lambda)^\beta - 1} \cdot v \cdot \Gamma(1 - 1/\beta)$$

and

$$\mathrm{E}[Y_W] = (1 + (s\lambda)^\beta)^{1/\beta} \cdot v \cdot \Gamma(1 - 1/\beta).$$

Proof. See Appendix A3.

The expected income of wage employed $\mathrm{E}[Y_W]$ obviously increases with both λ and s for given parameters β and v. It seems intuitively reasonable that the expected income of entrepreneurs $\mathrm{E}[Y_E]$ should also increase with λ but decrease with s. This conjecture is supported by numerical experiments for relevant values of λ, s and β. Moreover, suppose that for given s and β the demand for entrepreneurs increases. In a balanced labor market λ has to increase, which will increase the supply of entrepreneurs by an upward adjustment of

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[7] A positive random variable is *Fréchet* (β, v) -distributed with shape (variability) parameter $\beta > 0$ and scale (location) parameter $v > 0$, if its c.d.f. is $F(x) = e^{-(v/x)^\beta}$, $x > 0$. For the expected value to exist, it is required that $\beta > 1$, in which case it is $v \cdot \Gamma(1 - 1/\beta)$, where the gamma function is defined by $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$ for $x > 0$. Some basic properties of the Fréchet distribution are stated in Appendix A1. These properties form the basis for its mathematical tractability.

their relative incomes. Again numerical experiments for relevant parameter values indicate that $\mathrm{E}[Y_E]/\mathrm{E}[Y_W]$ then increases, and hence $\mathrm{E}[Y_E]$ increases relatively more than $\mathrm{E}[Y_W]$. If we assume that the demand for entrepreneurs is fixed then the product $s \cdot \lambda$ has to be constant by Proposition 2. It follows that if the utility adjustment factor, that is *s*, increases, then the expected income of entrepreneurs decreases, while that of wage employed remains constant.

We are now in a position to establish a fundamental inequality for our second measure of the relative expected income returns $\mathrm{E}[Y_E]/\mathrm{E}[Y_W]$.

Proposition 4. Consider our modified Lazear model with the same assumptions as in Proposition 2. Then the ratio between the expected incomes of entrepreneurs and wage employed is

$$\mathrm{E}[Y_E]/\mathrm{E}[Y_W] < 1/s .$$

Proof. See Appendix A4.

Although Fréchet-distributed skills are sufficient for this inequality to hold, it also holds for other skill distributions with a similar form. Considering the case $s = 1$, Lazear (2002) reports that the inequality is valid for gamma distributed skills for sufficiently low values of $\lambda > 1$. We have found that it also holds for Weibull-distributed skills under similar conditions.[8]

The proposition says that entrepreneurs on average will earn less than wage employed as long as entrepreneurship is not associated with any negative net non-pecuniary benefits, that is, if $s \geq 1$. However, entrepreneurs may on average earn more than wage employed if this is the case, that is, if $0 < s < 1$.

If there are no net non-pecuniary benefits, that is, if $s = 1$, then by Proposition 1, entrepreneurs as well as wage employed will on average earn more than they would do by making the opposite occupational choice; $\mu_E > 1$ and $\mu_W > 1$. By Proposition 4 this is fully consistent with the fact that entrepreneurs on average earn less than observationally similar wage employed. It is not necessary to introduce non-pecuniary benefits or income underreporting to explain this fact as scholars like Benz (2009) and Lechman (2015), for instance, have suggested. In this sense there is no entrepreneurship puzzle:

Corollary 1. Consider our modified Lazear model with the same assumptions as in Proposition 2. Then, for $s = 1$,

$$\mu_E > 1 \text{ and } \mu_W > 1$$

and

$$\mathrm{E}[Y_E] < \mathrm{E}[Y_W].$$

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[8] With Weibull-distributed skill profiles it is possible to derive closed-form expressions for the income distributions for self-employed and wage employed as well as their expected incomes. Preliminary analysis for $s = 1$ indicates that the expected income of self-employed is smaller or larger than that of wage employed depending on whether the market value of entrepreneurial talent λ is smaller or larger than some critical value $\bar{\lambda}$. This critical value decreases as the shape parameter β of the Weibull distribution increases.

Considering the modified Lazear model with the same assumptions as in Proposition 2, Proposition 5 in Appendix A5 provides closed-form expressions for the relative expected income returns for entrepreneurs and wage employed μ_E and μ_W, respectively.

Finally we will return to the self-selection bias associated with our second measure of income returns, given Fréchet-distributed skills. Recall that the bias b is defined as the ratio between our two measures of the relative expected income returns to entrepreneurship

$$b = \frac{\mathrm{E}[Y_E]}{\mathrm{E}[Y_W]} / \mu_E = \frac{\mathrm{E}[Z_E]}{\mathrm{E}[Y_W]}.$$

Since $\mathrm{E}[Y_E]/\mathrm{E}[Y_W] < 1/s$ by Proposition 4 and $1/s < \mu_E$ by Proposition 1, it always holds that $b < 1$.

Proposition 6. Consider our modified Lazear model with the same assumptions as in Proposition 2. Then, the self-selection bias b, as a function of the probability of entrepreneurship p_E, is

$$b(p_E) = \frac{(1-p_E)^{1/\beta} - (1-p_E)}{p_E},$$

where b is strictly decreasing from $1-1/\beta$ to 0 as p_E goes from 0 to 1 and also strictly decreasing in $\beta > 1$.

Proof. See Appendix A6.

The proposition says that the larger the fraction of entrepreneurs, the larger is the selection bias (smaller b).

3. The data

Our database – provided by Statistics Sweden – comprises individual time series data of all Swedish employed individuals from 2004 to 2008. Anyone working at least one hour per week in November each year, 4.4 million individuals in 2008, is considered employed the same year, implying that both full-time and part-time employees are included. Each individual is categorized as either wage employed or self-employed.[9] Co-owners of a business are included in the self-employment category. Individuals that combine wage employment and self-employment or co-owning are classified as wage employees if their income from wage employment is larger than the income from self-employment or co-owning of a firm and vice versa. Since combinations are rather common it should be noted that even a small income change might result in a reclassification of an individual from, say, self-employed to wage employed.

The income information corresponds to before-tax wage per year for wage employed and before-tax net business income per year for self-employed. Statistics Sweden multiplies reported business incomes by 1.6 in order to adjust for an observed tendency by business owners to report too low incomes. The incomes for those combining wage employment and self-employment include both types of incomes.

We will use the information about occupational status and income for three groups of working individuals that have a university degree as electrical engineer (Master of Science), architect

[9] All self-employed are owners of a registered incorporated or unincorporated business.

and college-educated non-performing artist.[10, 11] Since the purpose is to illustrate some empirical implications of our model specification, our calculations will be limited to the year 2008. The corresponding numbers of working individuals this year are: 13,100, 4,900 and 7,400, respectively.

One reason for choosing these groups is that type and length of education are key factors related to the choice between self-employment and wage employment according to Berglann et al. (2011), for example. Another reason is the conjecture that the skill profiles differ considerably regarding for instance scientific and artistic skills.

We will make a distinction between all self-employed and the subset of self-employed that hire at least one person, the latter to be denoted 1+. Furthermore, we will consider individuals that are wage employed or self-employed in 2008 as well as those that enter the labor market in 2008 and become either self-employed or wage employed. The individuals entering during a specific year are defined as those included in the database the same year but not the year before. It follows that we do not know if an enterer a certain year is a newcomer who has recently finalized the university education or someone who has a shorter or longer labor market experience but did not work at least one hour per week in November the preceding year.

Table 1 shows the average self-employment rates and relative income returns, measured as the ratios between the average incomes of self-employed and wage employed in 2008. Panel A concerns all individuals and panel B only those entering the labor market this year. As indicated in Appendix B, Table B1, the variation of the self-employment rates and income ratios across the years 2004-2008 is relatively small.

According to Panel A, the self-employment rates are much lower for electrical engineers than for architects and non-performing artists; 9.3 percent as compared to 26.0 and 27.4 percent, respectively. The corresponding percentages of self-employed hiring at least one individual are about half as high among electrical engineers and architects but considerably lower for non-performing artists.[12] A possible explanation might be that it is easier to find a job or at least a job matching the education, for electrical engineers.[13]

A comparison between the upper parts of Panel A and B shows that self-employment is more common among the enterers. For instance, about one third of the entering non-performing artists become self-employed. As one may expect, the percentages of self-employed that hire at least one individual are lower among enterers than among all employed. Assuming that those who hire at least one individual represent so-called start-up businesses, the percentage numbers imply that each educational group annually gives rise to around 15-20 new firms (see Appendix B, Table B1, Panel B).

Considering all self-employed, both Panel A and Panel B show that the income returns throughout are negative (below 100 percent). However, for the self-employed that hire at least

[10] The category “non-performing artists” covers visual artists, designers, media producers and craftspeople.

[11] According to Daghbashyan and Hårsman (2013), 10 percent of the electrical engineers, 20 percent of the architects and 25 percent of the non-performing artists combine self-employment and wage employment.

[12] The upper part of each panel compares all self-employed with all employed, and the lower part the subset of self-employed hiring at least one person with all wage employed

[13] As shown in table B1, the average income for wage employed electrical engineers is much higher than for wage employed architects and nonperforming artists.

Table 1. Percentage self-employed and relative income returns defined as average income of self-employed as a percentage of average income of wage employed in 2008 by education. The upper half of each panel includes all self-employed and the lower half only the subset of self-employed hiring at least one person (1+)

Variable	Electrical engineers	Architects	Non-performing artists
Panel A: All employed in 2008			
Percentage self-employed: all	9.3	26.0	27.4
Income returns	63.9	77.2	52.8
Percentage self-employed: 1+	4.3	14.2	5.2
Income returns	78.9	102.1	112.8
Panel B: All enterers in 2008			
Percentage self-employed: all	10.3	28.4	33.7
Income returns	50.5	34.7	53.1
Percentage self-employed: 1+	3.8	6.5	3.5
Income returns	119.5	93.7	105.1

one person, the returns are, with two exceptions, positive.[14] By way of example, an electrical engineer that enters the labor market will on average earn 19.5 percent more as self-employed than as wage employed provided that he hires at least one person. But an electrical engineer belonging to the category that also includes all own-account self-employed will on average earn 49.5 percent less. Our model provides some information about possible reasons for the income return differences between all self-employed and the self-employed that hires at least one individual. Since $\frac{\mathrm{E}[Y_E]}{\mathrm{E}[Y_W]} < 1/s$ by Proposition 4, and s is the utility adjustment factor, one explanation might be that the self-employed hiring others regard dis-benefits caused by risks, for instance, as more important, and positive benefits like being one's own boss as less important than the own-account self-employed.

Table B2 in Appendix B provides additional information about the observed income percentiles in 2008 for self-employed and wage employed engineers, architects and non-performing artists. The 5$^{\text{th}}$ income percentile is, with only one exception lower for self-employed than for wage employed. The pattern is somewhat less clear in the upper end of the income distribution but the 99$^{\text{th}}$ income percentiles indicate that all self-employed tend to be overrepresented provided they are hiring someone, and that the entering self-employed are overrepresented throughout. This pattern seems to be in line with the characterization of entrepreneurs as "stars and misfits" as suggested by Åstebro et al. (2011), for instance.

4. Results

The information about occupational status and income presented in Table 1 has been used to calibrate the parameters of the modified occupational choice model by applying the procedure described in Appendix A.7. Table 2 shows the calculated values of the counterfactual relative

[14] This is in line with lower average income for own-account self-employed compared to self-employed who hire others, see Parker (2009).

Table 2. Computed relative expected income returns to entrepreneurship μ_E and to wage employment μ_W and calibrated utility adjustment factor s, market value of entrepreneurial talent λ, shape parameter β and scale parameter v (in thousand SEK per year) by education. The upper half of each panel includes all self-employed and the lower half only the subset of self-employed hiring at least one person (1+)

	Parameter/variable	Electrical engineers	Architects	Non-performing artists
Panel A: All employed in 2008				
All self-employed vs. all wage employed	μ_E	0.98	1.30	1.03
	μ_W	1.78	1.48	2.23
	s	1.05	0.85	1.10
	λ	1.01	1.43	1.17
	β	2.95	2.70	2.23
	v	323	183	99
All self-employed hiring at least one person (1+) vs. all wage employed	μ_E	1.19	1.62	2.05
	μ_W	1.44	1.11	1.05
	s	0.85	0.65	0.5
	λ	1.21	1.70	2.09
	β	2.99	2.85	2.26
	v	332	202	113
Panel B: All enterers in 2008				
All self-employed vs. all wage employed	μ_E	0.80	0.55	1.27
	μ_W	2.26	3.24	2.27
	s	1.30	2.00	0.95
	λ	0.83	0.61	1.53
	β	2.82	3.00	1.88
	v	170	128	48
All self-employed hiring at least one person (1+) vs. all wage employed	μ_E	13.81	1.72	4.11
	μ_W	1.17	1.26	1.25
	s	0.08	0.60	0.25
	λ	14.30	1.77	4.21
	β	1.10	2.24	1.35
	v	15	109	27

income returns and the model parameters for each educational group. Panel A concerns all employed in 2008 and Panel B only those entering the labor market in 2008.

Defined in relation to the counterfactual income, the income returns to self-employment as well as to wage employment are positive among most sub-groups that are represented in the table. As shown by the upper part of Panel A, the returns to self-employment are -2 percent for electrical engineer, 30 percent for architects and 3 percent for non-performing artists. The self-employment returns among those that hire others are positive throughout and also higher than among all self-employed for both Panel A and B.[15] The income returns to wage employment are always positive and in most cases higher than the corresponding returns to self-employment. The results shown in the lower part of Panel B provide the most striking exceptions. It evidently seems to pay off much better – on average – to establish a firm and employ others than to become wage employed for the labor market beginners.[16]

As we clarify in Section 2, self-employment income return measures of the kind presented in Table 1 are biased by self-selection and this bias will increase will increase with the observed relative frequency of self-employed. Disregarding other influencing factors, the ratios between the return measures in Table 2 and Table 1 indicate the size of the bias.[17] To give an example, the ratios are 0.66 for electrical engineers hiring at least one person and 0.55 for the corresponding group of non-performing artists. Thus, Table 1 underestimates the income returns with 34 and 45 percent, respectively.

Table 2 also shows that the utility adjustment factor s is almost consistently less than 1, and consistently lower among the self-employed that hire others than among all self-employed. A possible explanation is that the trade-off between the hazards of entrepreneurship and the safety of wage employment is more important than the possible benefits of being one’s own boss for several categories of self-employed.

According to Lazear’s model, the market value of entrepreneurial talent λ adapts to ensure that the supply of entrepreneurs satisfies an exogenously given demand. The relationship between p_E, the probability of self-employment, and λ for given β and s,

$$p_E = \frac{(s\lambda)^\beta - 1}{(s\lambda)^\beta + 1},$$

can therefore be seen as equivalent to the entrepreneurial supply curve for a given population of individuals and the s and β-parameters as shift variables.

Using the parameter values for all entering electrical engineers, architects and non-performing artists, Figure 2 illustrates the corresponding the supply curves. As expected a higher market value of entrepreneurship increases the supply of self-employed. However, the supply consequences of the utility adjustment factor s are more striking. The intercepts of the supply curves are equal to the reciprocal values of the corresponding s-values. Since the latter value is about twice as high among architects as among non-performing artists it follows that their supply curve starts at a much lower level.

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[15] It can easily be verified, as suggested by Proposition 1, that the reported income returns to entrepreneurship are close to the averages between the corresponding values of λ and $1/s$.

[16] The high returns are rather uncertain, however, because of the small numbers of entering self-employed that hire others, see Table B1 in Appendix B.

[17] The results in Hårsman et al., 2018, applying an unmodified Lazear model, indicate that the self-selection bias will remain substantial even if one takes into account differences caused by gender, age, work experience and region.

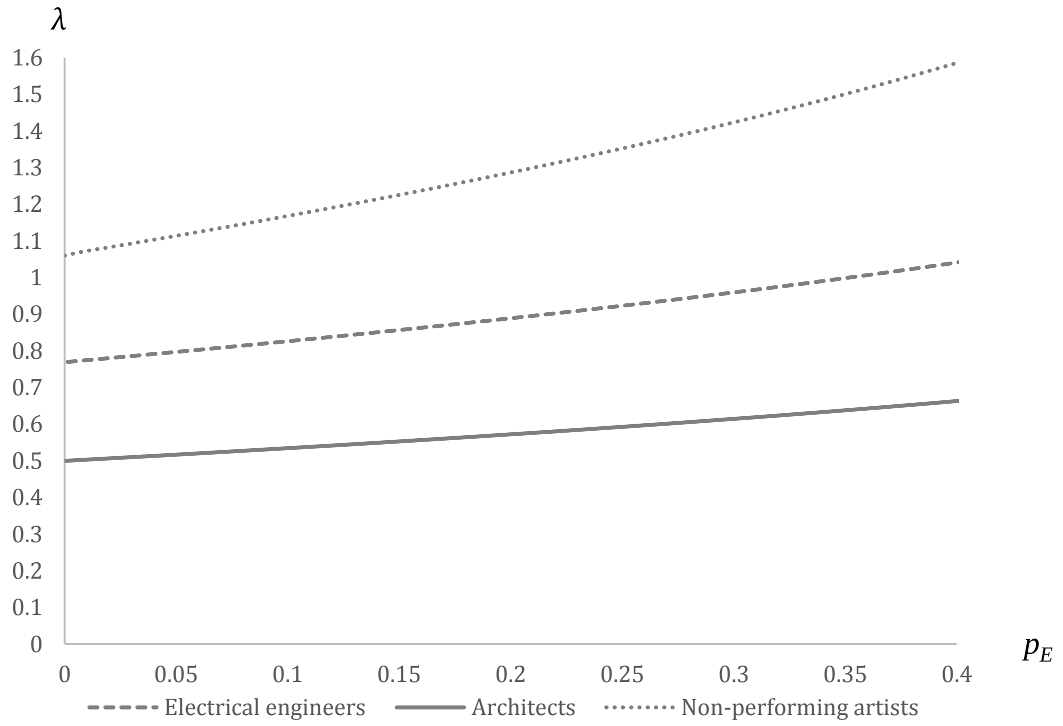

Figure 2. Supply curves for self-employment. The s -and β -values are chosen in accordance with Table 2, Panel B: all self-employed versus wage employed

As shown by Proposition 3 in Appendix A3, the income distributions for wage employed and entrepreneurs can be derived if the strengths of their skills are assumed to be Fréchet-distributed. Using the parameter values reported in Table 2, we have computed the 5th, 25th, 50th, 75th and 95th income percentiles and compared them with the corresponding observed income percentiles according to Appendix B, Table B2. Table 3 reports the ratio between each computed and observed income percentile.

Excluding the 5th percentile, the fit seems fairly good in Panel A. It is somewhat better for wage employed than for self-employed and about the same for all self-employed as for the self-employed hiring others. For enterers, that are Panel B, the calculated percentiles deviate more from the observed ones, especially among electrical engineers and non-performing artists.

It is impossible to assess to what extent the reported differences depend upon the applied model, the Fréchet assumption or the data. It should again be noted that there are only a small number of self-employed enterers in Panel B and their observed income distributions are thus particularly uncertain. We think, however, that fit is good enough to warrant further investigations. One obvious improvement would be to control for observable individual characteristics like age, gender and labor market region, and to use after-tax instead of before-tax incomes.[18] Another possibility is to use time series information and, for instance, consider the choice of wage employed to remain as wage employed or to become self-employed, and to include earlier experiences of wage and self-employment as additional characteristics.

[18] Underreporting of entrepreneurial incomes is another source of error even if Statistics Sweden has made an, admittedly rough, correction for that.

Table 3. The ratio between computed and observed income percentiles in the year 2008 by education and occupational status

Per-centiles	Electrical engineers				Architects				Non-performing artists			
	Self-employed		Wage employed		Self-employed		Wage employed		Self-employed		Wage employed	
	All	1+	All	1+	All	1+	All	1+	All	1+	All	1+
Panel A: All employed in 2008												
5th	-[a]	3.44	1.33	1.34	-[a]	3.31	1.81	1.91	-[a]	5.33	5.94	6.06
25th	1.40	1.10	0.94	0.95	1.79	1.05	0.82	0.85	5.94	1.16	0.94	0.96
50th	0.95	0.97	0.91	0.92	0.88	0.95	0.82	0.84	1.09	0.89	0.73	0.74
75th	0.82	0.91	0.97	0.97	0.78	0.90	0.90	0.91	0.63	0.85	0.80	0.80
95th	0.70	0.84	1.12	1.12	0.76	0.87	1.21	1.17	0.55	0.85	1.17	1.16
Panel B: All enterers in 2008												
5th	-[a]	5.28	2.07	0.15	-[a]	3.66	1.84	1.37	-[a]	-[a]	48	21
25th	4.13	2.06	0.90	0.09	2.27	1.32	1.07	0.87	6.33	1.43	0.88	0.45
50th	1.75	2.16	0.80	0.12	0.84	1.13	0.78	0.68	1.19	0.75	0.75	0.44
75th	1.19	1.70	0.96	0.25	0.46	0.80	0.84	0.81	0.70	0.76	0.69	0.48
95th	0.37	0.41	1.25	0.84	0.31	0.59	1.20	1.42	0.59	0.75	1.12	1.12

[a] Indicates that the observed income percentile is negative or equal to zero

5. Concluding remarks

Using a modified version Lazear's model of selection into entrepreneurship that includes a utility adjustment factor reflecting non-pecuniary net benefits, this paper analyzes two definitions of the returns to entrepreneurship – the returns relative to the counterfactual income and relative to the income of observationally similar wage employed. The theoretical part of the paper derives lower and upper bounds for the relative income returns defined in counterfactual terms. Adding the assumption that the skill profiles in a population are Fréchet-distributed, it is shown that income maximization will lead to lower expected earnings for entrepreneurs than for wage employed if the utility adjustment factor s is larger than or equal to 1. A comparison of our two income returns definitions provides a measure of the self-selection bias typically related to estimates of the average income difference between entrepreneurs and observationally similar wage employed. We demonstrate that this bias will increase with the observed percentages of entrepreneurs.

Data from the Swedish employment register is used to calibrate the model parameters and to assess the realism of including a utility adjustment factor and applying the Fréchet distribution. We do not think that results provide enough reasons to dismiss the model as unrealistic. It rather gains some support from a comparison of the observed gains some support from a comparison of the observed and the derived income distributions. It also gains some indirect support, since it allows, but does not require, the average income of self-employed to be lower than that of wage employed. Considering our empirical data this is the case for the three university-educated groups analyzed: electrical engineers, architects and non-performing artists as long as we include all own-account self-employed. This holds whether we consider all individuals that are wage employed or self-employed in 2008 or only those entering the labor market in 2008. However, comparing the average income of self-employed hiring at least one person with that of all wage employed, the relationship is reversed in four out of six cases. The calibrated values of the utility adjustment factor indicate that risk consideration might be as important as non-pecuniary benefits for the choice between self-employment and wage employment.

The main contributions of the paper are the derived upper and lower bounds for the relative income returns to entrepreneurship defined in counterfactual terms, the proposition that lower expected earnings for entrepreneurs than for wage employed is fully compatible with the Lazear model, and the derived measure of the self-selection bias inherent in estimates of the relative income returns to entrepreneurship defined in terms of observationally similar wage employed.

More generally, the paper underlines the importance of considering both the positive and the negative non-pecuniary aspects of entrepreneurship as well as making explicit assumptions about the distribution of skill profiles. If the occupational choice is solely based on income maximization ($s = 1$), then all entrepreneurs will earn more than they would do if they for some reason had been forced to make the opposite occupational choice. But whether utility maximization will result in higher or lower expected earnings for entrepreneurs than for wage employed, depends upon the assumptions made about non-pecuniary benefits and the distribution of skill profiles in the population. It follows that alternative assumptions about the utility adjustment and the skill distribution may also resolve the so called income puzzle of entrepreneurship. As an alternative to the Fréchet distribution, the Weibull distribution, that is another extreme value distribution being defined for positive values, is an obvious candidate to test.

We think that our specification of Lazear's model can be extended in several directions. One extension would be to investigate the possibility of estimating the market value of entrepreneurial talent as a function of individual characteristics like age, gender and their labor market region. Since the income returns to entrepreneurship and to wage employment should ensure that the supplies of entrepreneurs and specialists correspond to the demand, a more ambitious theoretical extension would be to develop a labor market model that allows for simultaneous estimation of the market value of entrepreneurial talent and the parameters of the skill profile distribution.[19]

Additionally, our approach opens up for analyzing measures of cognitive abilities in a new way. The empirical tests of Lazear's model reported by Aldén et al. (2017) and Hartog et al. (2010) can be complemented with estimations of the parameters of the Fréchet distribution, or other distributions of skill profiles, and the resulting implications for self-employment rates and earnings.

[19] Modeling trade, Eaton and Kortum (2002) use an estimation approach of this kind.

Appendix A. Additional propositions, proofs and parameter calibration

A1. Some basic properties of the Fréchet distribution

A continuous random variable $X>0$ is $\textit{Fréchet}(\beta,v)$-distributed with shape (variability) parameter $\beta>0$ and scale (location) parameter $v>0$, if its c.d.f. is

$$F(x)=e^{-(v/x)^{\beta}}, \quad x>0.$$

The expected value is $\mathrm{E}[X]=v\cdot\Gamma(1-1/\beta)$, provided that $\beta>1$, which is henceforth assumed, whereas the variance is $\mathrm{Var}[X]=v^2\cdot\left[\Gamma(1-2/\beta)-\left(\Gamma(1-1/\beta)\right)^2\right]$, provided that $\beta>2$. The expected value $\mathrm{E}[X]\to v$ and the variance $\mathrm{Var}[X]\to 0$ as $\beta\to\infty$. If X is $\textit{Fréchet}(\beta,v)$, then τX is $\textit{Fréchet}(\beta,\tau v)$ for any scalar $\tau>0$.

The following properties of the Fréchet distribution will be repeatedly used in the proofs.[20]

If X_1 and X_2 are statistically independent $\textit{Fréchet}(\beta,v_1)$ and $\textit{Fréchet}(\beta,v_2)$, respectively, then

$$p_1=\Pr[X_1>X_2]=\frac{v_1^{\beta}}{v_1^{\beta}+v_2^{\beta}}, \tag{1}$$

and

$$\hat{X}=\max\{X_1,X_2\},\ \hat{X}_1=(X_1|X_1>X_2) \text{ and } \hat{X}_2=(X_2|X_2>X_1) \text{ are all } \textit{Fréchet}(\beta,\hat{v}),$$

with

$$\hat{v}=(v_1^{\beta}+v_2^{\beta})^{1/\beta}. \tag{2}$$

A2. Proof of Proposition 2

Proof. The probability of a randomly drawn individual becoming wage employed is $p_W=\Pr[X_1>s\lambda X_2]+\Pr[X_2>s\lambda X_1]$. Since $s\lambda X_2$ and $s\lambda X_1$ are both $\textit{Fréchet}(\beta,s\lambda v)$ and X_1, and X_2 are statistically independent, it follows from eq. (1) that

$$p_W=2\frac{v^{\beta}}{(s\lambda v)^{\beta}+v^{\beta}}=\frac{2}{(s\lambda)^{\beta}+1},$$

and hence

[20] For proofs, see Corollary 3 in Mattsson et al. (2014), for example. By applying their general results, our present results can straightforwardly be extended to allow for positive statistical dependence between the random components by letting the joint c.d.f. of $\mathbf{X}=(X_1,X_2)$ be $F(x_1,x_2)=e^{-[(v_1/x_1)^{\rho\beta}+(v_2/x_2)^{\rho\beta}]^{1/\rho}}$, with $\rho\geq 1$. Obviously, for $\rho=1$ the random components X_1 and X_2 are statistically independent, while $\rho>1$ represents increasing positive statistical dependence between the components as $\rho\to\infty$.

$$p_E = 1 - p_W = \frac{(s\lambda)^\beta - 1}{(s\lambda)^\beta + 1}\,.^{21} \qquad (3)$$

A3. Proposition 3

In addition to the claims in Proposition 3, we also prove that the c.d.f. of the incomes of entrepreneurs and wage employed are, respectively,

$$F_{Y_E}(x) = \frac{2(s\lambda)^\beta \cdot e^{-(\lambda^\beta + s^{-\beta})\cdot(v/x)^\beta} - (1+(s\lambda)^\beta)\cdot e^{-2\lambda^\beta\cdot(v/x)^\beta}}{(s\lambda)^\beta - 1}, \quad x > 0,$$

and

$$F_{Y_W}(x) = e^{-(1+(s\lambda)^\beta)\cdot(v/x)^\beta}, \quad x > 0.$$

Proof. Note that since X_1 and X_2 are independent and identically distributed, all relationships are symmetric with respect to the two skills. Hence, it is no restriction to let the income of a wage employed be $Y_W = (X_1 | X_1 > s\lambda X_2)$ which by (2) is $Fréchet(\beta, \hat{v})$ with $\hat{v} = (v^\beta + (s\lambda v)^\beta)^{1/\beta} = (1+(s\lambda)^\beta)^{1/\beta} \cdot v$ from which its c.d.f. F_{Y_W} and expected value $\mathrm{E}[Y_W]$ immediately follow.

Similarly, it is no restriction to let the income of an entrepreneur be $Y_E = (\lambda X_1 | s\lambda X_1 > X_2 > X_1).$ Define the random variables $V = (\lambda X_1 | s\lambda X_1 > X_2) = (\lambda X_1 | \lambda X_1 > s^{-1} X_2)$ and $R = (\lambda X_1 | X_1 > X_2) = (\lambda X_1 | \lambda X_1 > \lambda X_2)$, which by (2) are $Fréchet(\beta, (\lambda^\beta + s^{-\beta})^{1/\beta} \cdot v)$ and $Fréchet(\beta, 2^{1/\beta} \lambda \cdot v)$, respectively. Evidently,

$$\Pr[s\lambda X_1 > X_2]\cdot F_V(x) = \Pr[s\lambda X_1 > X_2 > X_1]\cdot F_{Y_E}(x) + \Pr[X_1 > X_2]\cdot F_R(x),$$

where F_V, F_{Y_E} and F_R are the c.d.f.s of V, Y_E and R, respectively. By (1)

$$\Pr[s\lambda X_1 > X_2] = \frac{(s\lambda v)^\beta}{v^\beta + (s\lambda v)^\beta} = \frac{(s\lambda)^\beta}{1+(s\lambda)^\beta},$$

$$\Pr[X_1 > X_2] = \frac{1}{2},$$

and by (3)

[21] With a positive statistical dependence between the skills X_1 and X_2, represented by the parameter $\rho > 1$ as in the previous footnote, $p_E = [(s\lambda)^{\rho\beta} - 1]/[(s\lambda)^{\rho\beta} + 1].$ Hence, the probability of entrepreneurship approaches one as the positive statistical dependence between the skills approaches deterministic dependence, that is as $\rho \to \infty$.

$$\Pr[s\lambda X_1 > X_2 > X_1] = \frac{1}{2} P_E = \frac{1}{2} \cdot \frac{(s\lambda)^{\beta} - 1}{(s\lambda)^{\beta} + 1} \ .$$

Hence

$$F_{Y_E}(x) = \frac{\frac{(s\lambda)^{\beta}}{1+(s\lambda)^{\beta}} \cdot F_V(x) - \frac{1}{2} \cdot F_R(x)}{\frac{1}{2} \cdot \frac{(s\lambda)^{\beta}-1}{(s\lambda)^{\beta}+1}} = \frac{2(s\lambda)^{\beta} \cdot F_V(x) - (1+(s\lambda)^{\beta}) \cdot F_R(x)}{(s\lambda)^{\beta} - 1} \tag{4}$$

with $F_V(x) = e^{-(\lambda^{\beta}+s^{-\beta})\cdot(v/x)^{\beta}}$ and $F_R(x) = e^{-2\lambda^{\beta}\cdot(v/x)^{\beta}}$.

From (4) it follows that

$$\mathrm{E}[Y_E] = \frac{2(s\lambda)^{\beta} \cdot \mathrm{E}[V] - (1+(s\lambda)^{\beta}) \cdot \mathrm{E}[R]}{(s\lambda)^{\beta} - 1}$$

with $\mathrm{E}[V] = s^{-1}(1+(s\lambda)^{\beta})^{1/\beta} \cdot v \cdot \Gamma(1-1/\beta)$ and $\mathrm{E}[R] = 2^{1/\beta} \lambda \cdot v \cdot \Gamma(1-1/\beta)$.

A4. Proof of Proposition 4

Proof. Remember that $s \cdot \lambda > 1$ by assumption. The statement of the proposition, rewritten as $\mathrm{E}[Y_W] > s \cdot \mathrm{E}[Y_E]$, follows if the equality

$$\mathrm{E}[Y_W] = \alpha \cdot s \cdot \mathrm{E}[Y_E] + (1-\alpha) \cdot Q \tag{5}$$

is satisfied for some $\alpha \in (0,1)$ and $Q > \mathrm{E}[Y_W]$, since $\mathrm{E}[Y_W] \le s \cdot \mathrm{E}[Y_E]$ would then lead to a contradiction. Given the expressions for $\mathrm{E}[Y_W]$ and $\mathrm{E}[Y_E]$ from Proposition 3, the equality (5) is easily verified for $\alpha = \frac{(s\lambda)^{\beta}-1}{2(s\lambda)^{\beta}} \in (0,1)$ and $Q = 2^{1/\beta} s \cdot \lambda \cdot v \cdot \Gamma(1-1/\beta) > \mathrm{E}[Y_W]$.

A5. Proposition 5

Proposition 5. Consider Lazear's model with the same assumptions as in Proposition 2. The relative expected income returns to entrepreneurship are then

$$\mu_E = \frac{\mathrm{E}[Y_E]}{\mathrm{E}[Z_E]} = \frac{2^{1-1/\beta} s^{\beta-1} \lambda^{\beta} - \lambda \cdot (1+(s\lambda)^{\beta})^{1-1/\beta}}{(1+(s\lambda)^{\beta})^{1-1/\beta} - 2^{1-1/\beta}} ,$$

and the relative expected income returns to wage employment are

$$\mu_W = \frac{\mathrm{E}[Y_W]}{\mathrm{E}[Z_W]} = \frac{1}{\lambda \cdot (1+(s\lambda)^{\beta})^{1-1/\beta} - s^{\beta-1} \lambda^{\beta}} .$$

Proof. Since X_1 and X_2 are independent and identically distributed, it is no restriction to let $Z_E = (X_2 | s\lambda X_1 > X_2 > X_1)$. Define the random variables $V = (X_2 | X_2 > s\lambda X_1)$ and $R = (X_2 | X_2 > X_1)$, which by (2) are $\mathit{Fréchet}(\beta, (1+(s\lambda)^{\beta})^{1/\beta} \cdot v)$ and $\mathit{Fréchet}(\beta, 2^{1/\beta} \cdot v)$, respectively, and hence $\mathrm{E}[V] = (1+(s\lambda)^{\beta})^{1/\beta} \cdot v \cdot \Gamma(1-1/\beta)$ and $\mathrm{E}[R] = 2^{1/\beta} \cdot v \cdot \Gamma(1-1/\beta)$, respectively. Evidently,

$$\Pr[X_2 > X_1] \cdot \mathrm{E}[R] = \Pr[X_2 > s\lambda X_1] \cdot \mathrm{E}[V] + \Pr[s\lambda X_1 > X_2 > X_1] \cdot \mathrm{E}[Z_E].$$

By (1)

$$\Pr[X_2 > X_1] = \frac{1}{2},$$
$$\Pr[X_2 > s\lambda X_1] = \frac{1}{1+(s\lambda)^{\beta}},$$

and by (3)

$$\Pr[s\lambda X_1 > X_2 > X_1] = \frac{1}{2} P_E = \frac{1}{2} \cdot \frac{(s\lambda)^{\beta} - 1}{(s\lambda)^{\beta} + 1}.$$

Hence

$$\mathrm{E}[Z_E] = \frac{2^{1/\beta}(1+(s\lambda)^{\beta}) - 2(1+(s\lambda)^{\beta})^{1/\beta}}{(s\lambda)^{\beta} - 1} \cdot v \cdot \Gamma(1-1/\beta) ,$$

and together with the expression for $\mathrm{E}[Y_E]$ from Proposition 3 we obtain the required expression for μ_E.

Similarly, it is no restriction to let $Z_W = (\lambda X_1 \,|\, X_2 > s\lambda X_1)$. Define the random variable $V = (\lambda X_1 \,|\, s\lambda X_1 > X_2) = (\lambda X_1 \,|\, \lambda X_1 > s^{-1} X_2)$, which by (2) is *Fréchet*$(\beta, (\lambda^{\beta} + s^{-\beta})^{1/\beta} \cdot v)$, and hence $\mathrm{E}[V] = (\lambda^{\beta} + s^{-\beta})^{1/\beta} \cdot v \cdot \Gamma(1-1/\beta)$. Evidently,

$$\mathrm{E}[\lambda X_1] = \Pr[X_2 > s\lambda X_1] \cdot \mathrm{E}[Z_W] + \Pr[s\lambda X_1 > X_2] \cdot \mathrm{E}[V].$$

By (1)

$$\Pr[X_2 > s\lambda X_1] = \frac{1}{1+(s\lambda)^{\beta}},$$

and

$$\Pr[s\lambda X_1 > X_2] = \frac{(s\lambda)^{\beta}}{1+(s\lambda)^{\beta}}.$$

Since $\mathrm{E}[\lambda X_1] = \lambda \cdot v \cdot \Gamma(1-1/\beta)$ we get

$$\mathrm{E}[Z_W] = (\lambda \cdot (1+(s\lambda)^{\beta}) - (s\lambda)^{\beta} \cdot (\lambda^{\beta} + s^{-\beta})^{1/\beta}) \cdot v \cdot \Gamma(1-1/\beta),$$

and together with the expression for $\mathrm{E}[Y_W]$ from Proposition 3 we obtain the required expression for μ_W.

A6. Proof of Proposition 6

Proof. By definition, $b = \dfrac{\mathrm{E}[Y_E]}{\mathrm{E}[Y_W]}\Big/\mu_E = \dfrac{\mathrm{E}[Z_E]}{\mathrm{E}[Y_W]}$. Let $\varphi = (s\lambda)^\beta$. By Proposition 3 and 5, we have $b = \dfrac{2^{1/\beta}(1+\varphi)^{1-1/\beta} - 2}{\varphi - 1}$ and, since $\varphi = (1+p_E)/(1-p_E)$ by Proposition 2, b can be rewritten as a function of p_E as

$$b(p_E) = \frac{(1-p_E)^{1/\beta} - (1-p_E)}{p_E}.$$

Evidently $b = 0$ for $p_E = 1$ and by l'Hôpital's rule b goes to $1 - 1/\beta$ as p_E decreases to 0. We want to show that b as a function of p_E is strictly decreasing on the interval $(0,1)$.[22] It is somewhat simpler to prove the equivalent statement that b as a function of $p_W = 1 - p_E$ is strictly increasing on the interval $(0,1)$. Then

$$b(p_W) = \frac{p_W^{1/\beta} - p_W}{1 - p_W}$$

with the derivative

$$b'(p_W) = \frac{(1/\beta)p_W^{1/\beta - 1} + (1 - 1/\beta)p_W^{1/\beta} - 1}{(1-p_W)^2}.$$

Consider the numerator

$$c(p_W) = (1/\beta)p_W^{1/\beta - 1} + (1 - 1/\beta)p_W^{1/\beta} - 1$$

and note that $c(1) = 0$. Hence, $b'(p_W) > 0$ for all $p_W \in (0,1)$ if $c'(p_W) < 0$ for all $p_W \in (0,1)$. Recalling that $\beta > 1$, this is indeed the case since,

$$c'(p_W) = -\tfrac{1}{\beta}(1 - \tfrac{1}{\beta})(1 - p_W)p_W^{1/\beta - 2} < 0.$$

Finally, it is easily verified that b as function of $\beta > 1$ is strictly decreasing.

A7. Applied procedure for parameter calibration

To apply the modified Lazear model as specified in Proposition 2, four parameters need to be calibrated against empirical data: s, λ, β and v. It is assumed that data are available for the

[22] The authors thank Jörgen Weibull for suggesting this proof.

relative frequency of entrepreneurs (self-employed) $\bar{p}_E$ as well as for the p-percentiles of the income distributions $\bar{Y}_p^E$ and $\bar{Y}_p^W$, $p \in \{0.05, 0.25, 0.5, 0.75, 0.95\}$, and the average incomes $\bar{Y}_E$ and $\bar{Y}_W$ of entrepreneurs and wage employed, respectively. We will use $\bar{p}_E$ as our observation for p_E, and $\bar{Y}_E$ and $\bar{Y}_W$ as our observations for $\mathrm{E}[Y_E]$ and $\mathrm{E}[Y_W]$, respectively.

Let $\varphi \equiv (s\lambda)^{\beta}$. By Proposition 2

$$\varphi = \frac{1+\bar{p}_E}{1-\bar{p}_E}$$

and by Proposition 3

$$\frac{\bar{Y}_E}{\bar{Y}_W} = \frac{1}{s}\cdot\frac{2\varphi - 2^{1/\beta}\varphi^{1/\beta}(1+\varphi)^{1-1/\beta}}{\varphi-1}.$$

The last equation can be solved for β as a function of $s \in (0, \bar{Y}_W/\bar{Y}_E)$

$$\beta(s) = \frac{\ln(2\varphi)-\ln(1+\varphi)}{\ln[2\varphi - s\cdot(\bar{Y}_E/\bar{Y}_W)\cdot(\varphi-1)] - \ln(1+\varphi)},$$

and hence λ as a function of s is

$$\lambda(s) = \frac{\varphi^{1/\beta(s)}}{s} = \frac{1}{s}\cdot\left(\frac{1+p_E}{1-p_E}\right)^{1/\beta(s)}$$

and by Proposition 3, v as a function of s is

$$v(s) = \frac{\bar{Y}_W}{(1+\varphi)^{1/\beta(s)}\cdot\Gamma(1-1/\beta(s))}.$$

Next, the p-percentiles $Y_p^E(s)$ and $Y_p^W(s)$ of the income distributions F_{Y_E} and F_{Y_W}, as specified in the proof of Proposition 3 in Appendix A3, are derived from the equations $F_{Y_E}(Y_p^E(s)) = p$ and $F_{Y_W}(Y_p^W(s)) = p$, given the parameters $\beta(s), \lambda(s), v(s)$ and s, for $p \in \{0.05, 0.25, 0.5, 0.75, 0.95\}$. The equations for the percentiles for wage employed can be solved analytically

$$Y_p^W(s) = v(s)\left(\frac{1+\varphi}{-\ln p}\right)^{1/\beta(s)},$$

while the equations for the percentiles for entrepreneurs $Y_p^E(s)$ have to be solved numerically.

Finally, s is determined so that the sum of the quadratic deviations between the p-percentiles of the observed and computed income distributions of entrepreneurs and wage employed is minimized: [23]

$$\min_{s \in (0, \bar{Y}_W / \bar{Y}_E)} \sum_{p \in \{0.05,\, 0.25,\, 0.5,\, 0.75,\, 0.95\}} \left[(Y_p^E(s) - \bar{Y}_p^E)^2 + (Y_p^W(s) - \bar{Y}_p^W)^2 \right].$$

[23] From a general point of view, the calibrated parameters correspond to regression estimates. The averages of the observed and computed incomes will be equal and the sum of squared differences between observed and computed income percentiles is minimized.

Appendix B. Additional data

Table B1. Number of individuals *n*, average income $\bar{Y}$ and median income *m* by education, year and occupational status. Annual income in thousand SEK

Panel A: All wage employed, all self-employed and the subset of self-employed hiring at least one person (1+)

Occupational status	Electrical engineers			Architects			Non-performing artists		
	n	$\bar{Y}$	m	n	$\bar{Y}$	m	n	$\bar{Y}$	m
2004									
Wage employed	11210	518	479	3223	330	329	4336	218	208
Self-employed: all	1000	340	309	1245	257	252	1887	123	86
Self-employed: 1+	454	420	391	564	341	334	235	262	251
2005									
Wage employed	11325	528	487	3322	337	337	4580	227	213
Self-employed: all	1066	349	326	1246	272	273	1939	133	88
Self-employed: 1+	488	431	405	561	367	353	263	248	237
2006									
Wage employed	11669	539	496	3416	355	346	4888	236	224
Self-employed: all	1091	364	344	1284	285	286	1935	146	108
Self-employed: 1+	470	422	411	584	372	362	260	252	240
2007									
Wage employed	11909	557	508	3599	364	356	5230	238	231
Self-employed: all	1122	358	340	1242	299	288	1878	130	102
Self-employed: 1+	476	443	432	569	403	370	253	259	259
2008									
Wage employed	11877	576	524	3657	377	370	5367	251	249
Self-employed: all	1211	368	361	1287	291	304	2021	133	108
Self-employed: 1+	533	454	438	603	385	377	292	283	283

Table B1. Continued.

Panel B: All enterers becoming wage employed, self-employed and the subset of self-employed hiring at least one person (1+)

Occupational status	Electrical engineers			Architects			Non-performing artists		
	n	$\overline{Y}$	m	n	$\overline{Y}$	m	n	$\overline{Y}$	m
2005									
Wage employed	709	248	251	292	182	181	577	134	125
Self-employed: all	69	162	104	95	104	96	194	55	40
Self-employed: 1+	19	179	150	18	190	221	22	79	56
2006									
Wage employed	747	277	267	267	193	193	609	131	114
Self-employed: all	73	140	100	92	142	82	187	58	44
Self-employed: 1+	17	187	140	18	172	166	20	79	77
2007									
Wage employed	625	305	300	262	218	232	587	155	143
Self-employed: all	70	141	84	101	133	84	208	55	43
Self-employed: 1+	15	169	100	19	222	140	14	97	111
2008									
Wage employed	503	314	322	245	244	262	524	162	139
Self-employed: all	58	159	84	97	85	94	266	86	62
Self-employed: 1+	20	375	113	17	229	180	19	170	171

Table B2. Observed income percentiles in the year 2008 by education and occupational status (in thousand SEK per year)

Per-centiles	Electrical engineers			Architects			Non-performing artists		
	Self-employed		Wage employed	Self-employed		Wage employed	Self-employed		Wage employed
	All	1+		All	1+		All	1+	
Panel A: All employed in 2008									
5th	0	87	219	0	75	97	-100	30	16
25th	209	329	401	126	288	287	16	176	142
50th	361	438	524	304	377	370	108	283	249
75th	505	563	664	424	483	465	240	381	342
95th	822	845	1 028	625	707	659	431	590	506
99th	1091	1106	1776	981	1021	914	638	814	734
Panel B: All enterers in 2008									
5th	-17	18	74	-20	35	68	-33	-9	1
25th	30	77	224	30	125	152	9	63	82
50th	84	113	322	94	180	262	62	171	139
75th	150	239	366	207	328	326	141	254	242
95th	682	2 460	518	424	691	403	286	540	373
99th	4397	4397	863	750	691	573	499	540	422

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