Magnetic circuits

Dealing with magnets (electric or permanent) requires the use of a formalism bearing strong resemblance to Ohm-Kirchhoff's treatment of DC-circuits. Typical problems addressed are:
What will be the lifting force of this electromagnet?
What inductive voltage will this permanent magnet cause when passing by this coil?
How should this relay be designed?

Basic setup

We first start with a setup having a ferromagnetic material in the shape of a formalized horseshoe (fig 1). This is the electromagnet and has a cross section S_1, a relative magnetic permeability μ_{r1} and a medium magnet path length L_1 described by the dashed, red line through material no 1. Around one leg of this material a coil with N turns carrying current I is wrapped.
The object to be lifted is made from ferromagnetic material no 2 characterized in the same way. In the first setup we treat the permeabilities as constants meaning we do not apply currents strong enough to cause permanent magnetization.
There are always also more or less pronounced "airgaps" between the two materials. These can be either rust, dirt, oil films or actual air, the important part is that they are not ferromagnetic.

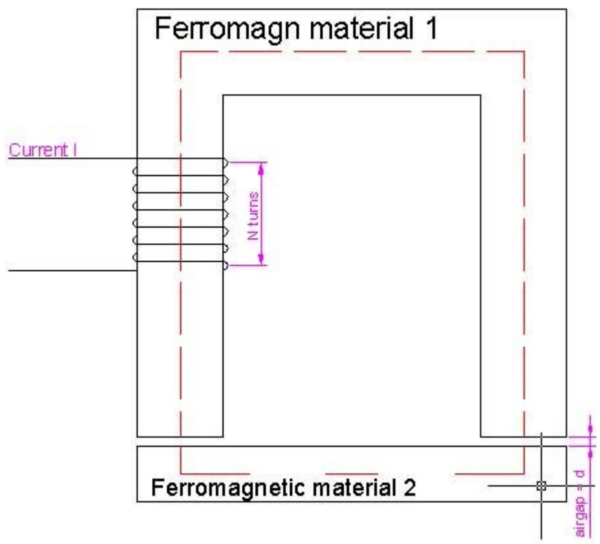


Fig 1, Basic setup

Analysis of electromagnet

The analysis will depend upon Ampéres law

$$(1) \qquad \oint_{path} \vec{H} \cdot d\vec{l} = \sum_{components} Hdl = \sum_{insidepath} I$$

and that the divergence of B is zero, meaning that no B-lines start or stop anywhere and that the total magnetic flux is conserved. We also utilize the fact found previously that the magnetic flux follow ferromagnetic materials whenever possible.
Applying (1) to our case will give

$$(2) \qquad H_{f1}L_1 + H_{f2}L_2 + H_{air}2d = NI$$

where the integration is performed along the red dashed line.
The H and B fields are related via the constituitive relations

$$(3) \qquad B_{air} = \mu_0 H_{air} \qquad B_{fi} = \mu_0 \mu_r H_{fi} \quad where \quad i = 1{,}2$$

Paramagnetic and diamagnetic effects are neglected.
Conservation of magnetic flux gives us

$$(4) \qquad \Phi_M = B_1 S_1 = B_{air} S_1 = B_2 S_2$$

As can be seen there are a lot of geometric approximations done here, as well as the approximation that B is constant over the cross section. These approximations are validated by the fact that results achieved are in very good agreement with experimental data.
Combining 2 and 3 gives

$$(5) \qquad \frac{B_{f1}}{\mu_0\mu_{r1}}L_1 + \frac{B_{f2}}{\mu_0\mu_{r2}}L_2 + \frac{B_{air}}{\mu_0}2d = NI$$

Combining 4 and 5 we can solve for any of the B's. From a magnetic lifting point of view the field in the air gap is most important so we start with that

$$(6) \qquad \frac{B_{air}}{\mu_0\mu_{r1}}L_1 + \frac{S_1}{S_2}\frac{B_{air}}{\mu_0\mu_{r2}}L_2 + \frac{B_{air}}{\mu_0}2d = NI$$

Which can be rewritten by (strangely) taking the magnetic flux $B_{air}S_1$ out of the left member in eq 6.

$$(7) \qquad B_{air}S_1\left(\frac{L_1}{\mu_0\mu_{r1}S_1} + \frac{L_2}{\mu_0\mu_{r2}S_2} + \frac{2d}{\mu_0 S_1}\right) = NI$$

The terms in the parenthesis are called reluctancies, R_m, of the components, and hence 7 can be written

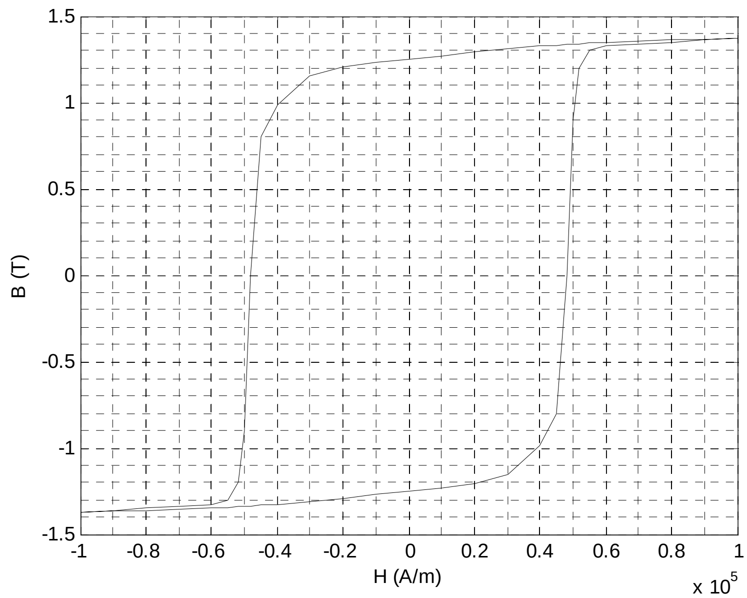
$$(8) \quad \Phi_M\left(R_1 + R_2 + R_{air}\right) = \Phi_M R_{tot} = NI$$

Which very much look like resistors coupled in series treated by Ohm's law. Note that NI corresponds to the voltage (= what drives the action), Φ_M corresponds to the current (= the action) and R_m corresponds to resistance (= what controls the action).
A nice fact is of course that reluctancies coupled in series are summed, as are resistancies. If reluctancies are coupled parallel the summation rule is the same as for parallel resistors, which can be proved exactly the same way as for resistors. Parallel reluctancies only live in text books however.
The dependencies of R_m on length, cross section area and permeability (corresponds to conductivity) are also the same as for resistors.
Note also that in order for the reluctancies to be comparable in magnitude, the total air gap length should be in the order L/μ_r. If we want a strong field we should try to minimize the air gaps! A horse-shoe is better than a rod, and the horse-shoe's both legs should be used.
Electromagnets are almost invariably done like horse-shoes but normally very generalized horse-shoes. A common design is to have a inner cylinder of ferromagnetic material surrounded by the coil. Outside this we have a cylinder of nonmagnetic metal (=very high reluctance) and outside this we have a cylinder of ferromagnetic material. On top we have a plate of ferromagnetic material. If we now put the thing we want to lift (metal waste as an example) under it we have a closed circuit. From the outside it just looks as an all metal cylinder. A horse-shoe no horse ever would or could wear.

Analysis of permanent magnets

A permanent magnet is a piece of ferromagnetic material where the primary crystal regions are irreversibly oriented. This means that H and B are related via the hysteresis curve instead of the permeability. One could in principle talk about μr also in this case, but it would normally have a negative value (!)
A typical hysteresis curve for a permanent magnet material (AlNiCo6) is shown on the next page:

This describes the material relation between H and B once the material has been magnetized to saturation. Note that there is no way to teach origo again once permanent magnetization has set in.
Let us use this material for the horse shoe in the standard setup as in the previous figure. The item to lift (no 2) is the same as previously.
No current in the coil this time.
The idea is to use a graphic method where the reformulated eq 6 generates a straight line that can be plotted into the hysteresis diagram.



Eq 5 now becomes

(9) $$H_{f1}L_1 + \frac{B_{f2}L_2}{\mu_0\mu_{r2}} + \frac{2dB_{air}}{\mu_0} = 0$$

Eq 3 is still valid for air and the item to be lifted, and eq 4 is valid throughout:

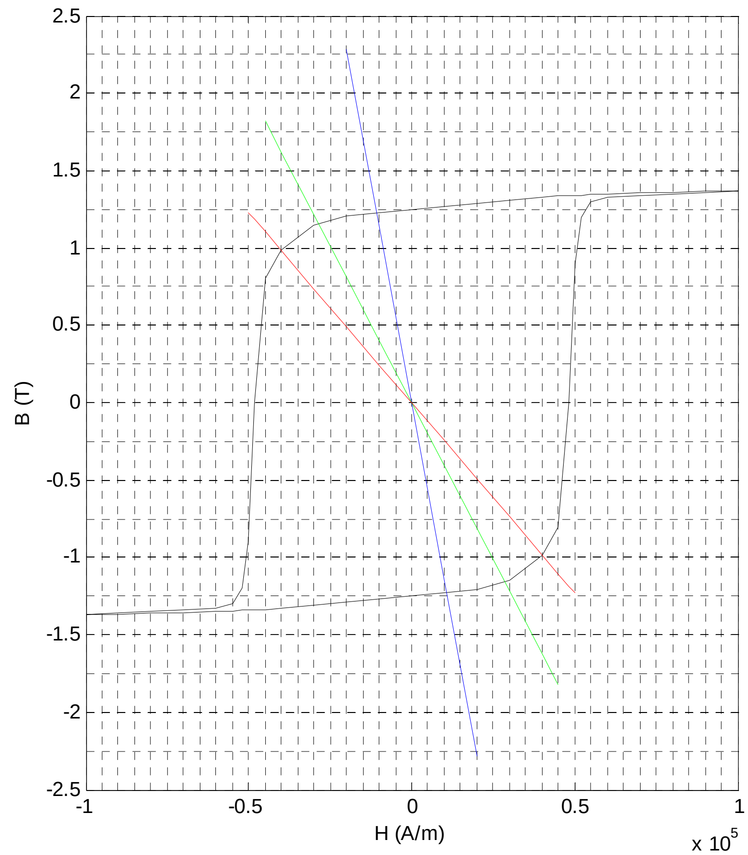
(10) $$H_{f1}L_1 + \frac{S_1B_{f1}L_2}{\mu_0\mu_{r2}S_2} + \frac{2dS_1B_{f1}}{\mu_0S_1} = 0 \Rightarrow$$

(11) $$B_{f1} = \frac{-L_1}{\frac{S_1L_2}{\mu_0\mu_{r2}S_2} + \frac{2d}{\mu_0}} H_{f1} = \frac{-\mu_0\mu_{r2}S_2L_1}{S_1L_2 + 2d\mu_{r2}S_2} H_{f1}$$

Some letters to write but it is just a straight line through origo with negative coefficient of direction. Observe that if the air gaps grow the second term in the denominator rapidly makes the direction coefficient drop.
We plot this line and read the B-value at intersection.
Using eq 4 we then fins the B-value in the air gap. Because of the nonlinear behavior of the curve it is hard to predict whar will happen.

Let us see how this works in three cases:

- A/ μr2=500, S1=S2, L1=200 mm, L2=100 mm, d=1 mm, blue line
- B/ same except d=3 mm, green line
- C/ same except d=5 mm, red line



A gives B= 1,2 T
B gives B= 1,16 T
C gives B= 0,99 T
But for larger values of d the drop will get rapid.
The bar magnet would correspond roughly to d = L1 which gives B= 0,02T, which is significantly smaller.
Remember that the attractive force between magnet and object is given by

$$F = \frac{B_{air}{}^{2}}{2\mu_0}$$

Seemingly independent of d!
The reason magnets and iron objects behave the way they do (sudden snap on when object gets close enough) is the change in B.