Suggested solutions to exam in Applied Electromagnetism 081023

1

Coordinates: x = observation direction; y = along the wires; z = vertical
Wires at (0,0,d/2) and (0,0,-d/2) where d = 10 mm
Observation at (L,0,0) where L = 1,0 m
Because of symmetry only z components survive
From one wire we get

$$E_{one} = \frac{\lambda}{2\pi\varepsilon_0\sqrt{L^2 + \left(d^2/4\right)}}$$

Comp in z direction is obtained by multiplying with $\dfrac{d/2}{\sqrt{L^2 + \left(d^2/4\right)}}$ so

$$E_{tot} = \frac{2d\lambda/2}{2\pi\varepsilon_0\left(L^2 + \left(d^2/4\right)\right)} = \frac{d}{\sqrt{L^2 + \left(d^2/4\right)}} E_{one} \approx 0{,}12\ \mathrm{V/m}$$

2
It is an order of magnitude problem. Use Biot Savart

$$B = \frac{\mu_0 I d\vec{s} \times \vec{r}}{4\pi r^3} \approx \frac{\mu_0 \Delta Q \Delta s}{4\pi \Delta t r^2} = \frac{\mu_0 v \Delta Q}{4\pi r^2} \approx 10^{-16} T$$

Not a huge field…
3-4
It is a magnetic circuit

$$N_1 I = H_{ferro} 2\pi R_m + H_{air} 2d = B_{air}\left(\frac{2\pi R_{\mathrm{M}}}{\mu_0 \mu_r} + \frac{2d}{\mu_0}\right) \Rightarrow B = \frac{\mu_0 \mu_r N_1 I}{2\pi R_m + 2d\mu_r} \Rightarrow$$

$$M_{12} = \frac{N_2 \Phi_{12}}{I_1} = \frac{\mu_0 \mu_r N_1 N_2 S}{2\pi R_m + 2d\mu_r} = 0{,}13\,\mathrm{mH}$$

Now $U_2 = M_{12}\dfrac{dI_1}{dt} = M_{12} 2\pi f I_{\max} \sin\left(2\pi f t\right)$

giving $fI_{\max} = \dfrac{U_2}{2\pi M_{12}} = 6{,}1\,\mathrm{A/s}$

At 50 Hz (as an example) a current with max120 mA is needed
5
At the nail the field is (without the nail)

$E_{down} = \dfrac{p}{4\pi\varepsilon_0 L^3}$ inducing a polarization in the nail that just compensates this inside the nail giving

$$\sigma_{nail} = \varepsilon_0 E_{down}$$

The dipole moment of the nail will then be $p_{nail} = ah^2\sigma_{nail} = \dfrac{ah^2 p}{4\pi L^3}$

..and the field from this back on the ground

$$E_{ground} = \frac{p_{nail}}{4\pi\varepsilon_0 L^3} = \frac{pah^2}{16\pi^2\varepsilon_0 L^6} = 0{,}71\,\mathrm{V/m}$$

6

$$E = \frac{\lambda}{2\pi\varepsilon_0\varepsilon_r x} + \frac{\lambda}{2\pi\varepsilon_0\varepsilon_r (4r - x)} \Rightarrow U = \int_r^{3r} E dx = \frac{Q}{2\pi\varepsilon_0\varepsilon_r l}\left[\ln x - \ln\left(4r - x\right)\right]_r^{3r} = \frac{Q}{2\pi\varepsilon_0\varepsilon_r l} 2\ln 3$$

$$C/l = \frac{Q}{Ul} = \frac{\pi\varepsilon_0\varepsilon_r}{\ln 3}$$