Suggested solutions to exam in Applied Electromagnetism 091023

1

Assuming charge Q in one tip (center of curvature in x=0) and –Q (in x=d) in the other, gives the combined potential

$$U(x)=\frac{Q}{4\pi\varepsilon_0 x}+\frac{Q}{4\pi\varepsilon_0 (x-d)}\Rightarrow U_{+-}=U(d-r)-U(r)=\frac{Qd}{2\pi\varepsilon_0 r(d-r)}\Rightarrow C=\frac{Q}{U_{+-}}=\frac{2\pi\varepsilon_0 r(d-r)}{d}$$

Vilket med insatta värden ger ca 6 fF

Assuming, homogenous field between gives at least a factor 10 wrong.

2

The field from the ring is

$$E_{ring}=\frac{2\pi R\lambda x}{4\pi\varepsilon_0 (x^2+R^2)^{3/2}}=\frac{Qx}{4\pi\varepsilon_0 (x^2+R^2)^{3/2}}$$ where R is radius of the ring, x is distance along axis

Condition of deviation 10% gives

$$0.1=\frac{E_{po\,\mathrm{int}}-E_{ring}}{E_{po\,\mathrm{int}}}=1-\frac{x^3}{\left(x^2+R^2\right)^{3/2}}\Rightarrow 0.9^{2/3}=\frac{x^2}{x^2+R^2}\Rightarrow x=3.7R$$

3

Newton's second gives

$$F_x=\frac{dp_x}{dt}=qE+\frac{q}{m}p_yB,\quad F_y=\frac{dp_y}{dt}=-\frac{q}{m}p_xB,\quad F_z=0$$ Combining gives

$$\frac{d^2p_x}{dt^2}=\frac{qB}{m}\frac{dp_y}{dt}=-\frac{q^2B^2}{m^2}p_x=-\omega^2p_x$$

This has the general solution

$$p_x=A\sin(\omega t)+B\cos(\omega t)\quad where\quad p_x(t=0)=0\Rightarrow B=0$$

$$p_y=\frac{1}{\omega}\frac{dp_x}{dt}-m\frac{E}{B}=A\cos\omega t-m\frac{E}{B}=0\quad when\ t=0\Rightarrow A=\frac{mE}{B}$$

Integrating p_x and p_y gives

$$x=\frac{E}{\omega B}(1-\cos\omega t)\quad and\ \ y=\frac{E}{\omega B}\sin\omega t-\frac{E}{B}t$$

So: circles that are gradually moving in the y direction

5

Horisontal field component can be derived the same way as field from electrical dipole

$$B_{horiz}=\frac{\mu_0 I\cos\alpha}{2\pi}\left[\frac{1}{\sqrt{r^2+\left(d/2\right)^2+rd\cos\beta}}-\frac{1}{\sqrt{r^2+(d/2)^2-rd\cos\beta}}\right]\quad where\quad \alpha+\beta=90°$$

$$\Rightarrow B_{horiz}\propto\frac{\cos\alpha\sin\alpha}{r^2}=\frac{\cos^3\alpha\sin\alpha}{h^2}$$ vilken har maximum för α = 30°

..at 5,7m distance from "just under"

6

Potential is not defined for one wire but for two it will be:

$$E=\frac{\lambda}{2\pi\varepsilon_0 x}-\frac{\lambda}{2\pi\varepsilon_0 (x+d)}\approx\frac{\lambda d}{2\pi\varepsilon_0 x^2}\Rightarrow U=\frac{\lambda d}{2\pi\varepsilon_0 x}$$

You cannot start integrating U directly as you will subtract infinities in that case.