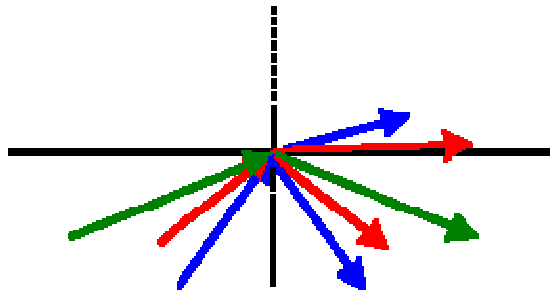
Fiber optics

Introduction



An optical fiber is a component with the purpose of transferring large amounts of information in a short time. The transfer makes use of laser light, directed into a material with a higher index of refraction than its surroundings. The conduction of light is based on total reflection, i.e. when light is to be refracted through a surface and when it is coming from a medium with a higher index of refraction than the medium it is going to, it does not give rise to a refracted beam if the angle of incidence is large enough. See figure. The angle of incidence of the red beam is so large that the refraction angle becomes nearly 90^{o}. It cannot easily become larger than 90^{o}, so for larger angles of incidence (green in the figure) all the light will be reflected. The idea is now to construct a thread from a material with a higher index (core) surrounded by material with a somewhat lower index (cladding). The method of conducting light will now work in the core, even if it is bent slightly.

There are many advantages using fibers for transfer of information instead of different electrical alternatives (e.g. twin cables and coaxial cables):

- It is possible to send more units of information (e.g. bits) per second in a fiber because a light wave has a much higher frequency than electrical signals. (Hundreds of THz instead of a few GHz). On the other hand, the signal does not arrive faster, since information in an ordinary coaxial cable also travels with the speed of light.
- The fiber does not leak out any signal which can be intercepted in any reasonable way, which is what electrical cables do. Conversely, fibers are not particularly sensitive to different kinds of disturbances from the outside. EMP and strong magnetic fields from transformers leave the optical signal essentially unaffected.
- The fiber is lighter than most other materials, which can be useful for some applications.
- The raw material to the optical fiber is crushed quartz (i.e. sea sand), available in large quantities and therefore cheap.

Background

In order to conduct light through a fiber it is necessary that the angle of incidence at the core/cladding interface be all the time larger than the limiting angle for total reflection. This is given by the requirement that the angle of refraction is 90^{o} (it can hardly be larger):

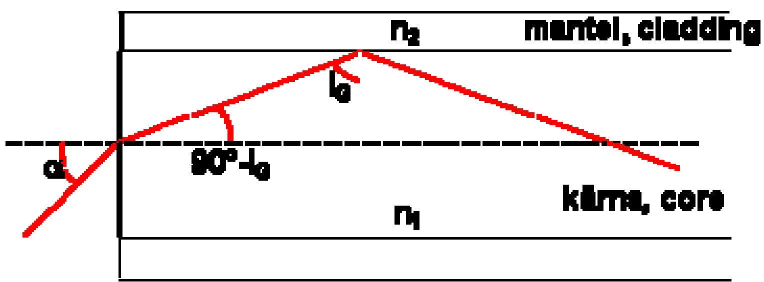
$$n_1 \sin i_G = n_2 \sin 90^o = n_2 \quad \Rightarrow \quad \sin i_G = \frac{n_2}{n_1}$$

For this to happen inside the fiber, certain requirements are put on the angle at which the light is led into the fiber. See figure. One obtains a largest value of α, corresponding to the smallest value of i_G , which leads to total reflection. The sinus of that angle is so important that it has got a name of its own: Numerical aperture or NA:

$$NA = \sin\alpha = n_1 \sin(90^o - i_G) = n_1 \cos i_G = n_1\sqrt{1-\sin^2 i_G} = n_1\sqrt{1-n_2{}^2/n_1{}^2} = \sqrt{n_1{}^2 - n_2{}^2}$$

Putting n_2 equal to n_1-Δn and realizing that Δn is small, the last part can be rewritten as:

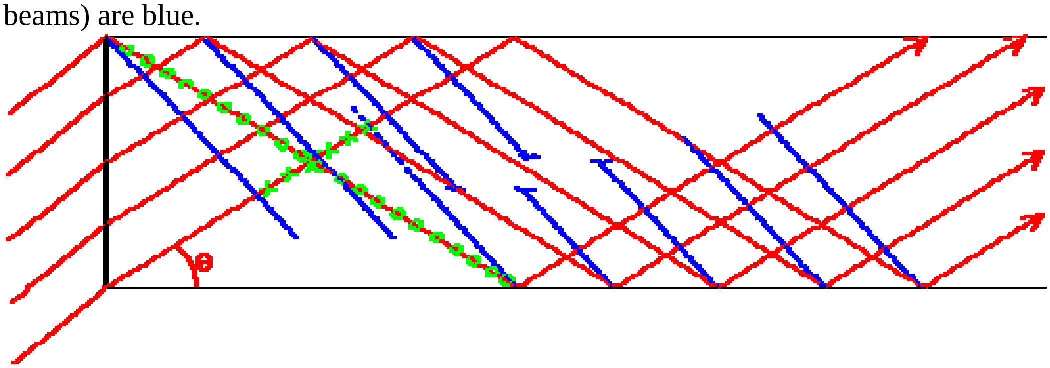
$$NA = \sqrt{n_1{}^2 - \left(n_1{}^2 - 2\Delta n n_1 - (\Delta n)^2\right)} \approx \sqrt{2\Delta n n_1}$$



NA gives thus the angular range within which the beams must lie in order to be able to be transported further ahead.

This now has to be combined with yet another condition, a slightly more cumbersome one: The wave nature of light has to be considered since the fiber is so small. The wavefronts must remain unbroken.

We start by considering a highly simplified situation: Look now at the next figure. We see waves falling in at a certain angle. The beams are red and get the angle θ inside the fiber, while the wavefronts belonging to the beams (always perpendicular to the beams) are blue.



For the light to be led into the fiber, the extensions of the wavefronts before and after the first reflection must coincide, i.e. not like in the figure. (The dashed extension of a wavefront does not fit in). A condition for them to do it, is that the difference of the lengths marked by green crosses and green rings be a multiple of a wavelength.

We must realize now (not wanting to perform the derivation in its entirety) that the angles for which the above condition is fulfilled are more and more densely located as the size of the fiber increases. If the fiber is extremely small, there might not exist any such angles.

This is something we want to make use of in order to construct a so-called single-mode fiber:
First of all: The angle $\theta = 0$ is always an allowed path, or mode, for the light, since no problems with wavefronts will occur in that case.
Choose now the diameter of the fiber core so that the first angle fulfilling the condition ”green difference” = a multiple of wavelengths, is larger than NA. This means that there is only one way the light can enter the fiber, which is highly advantageous, since the presence of many allowed entrance angles would lead to many conceivable transit times for a pulse through the fiber, leading to pulse-widening (i.e. –destruction). With the present choice of materials, the core diameters of such fibers will be around 3-8 μm. The cladding is then made much thicker in order to give mechanical stability to the fiber.

The damping of single-mode fibers is today about 0.1 dB/km. After 300 km (Stockholm-Karlstad), the intensity has gone down by a factor of 1000! This corresponds (as a curiosity) to the fact that if the sea were as transparent as the fiber, one would be able to look down through the surface at the Mariana Trench and see the bottom essentially without damping.

The same condition can be obtained starting from the waveguide, using Fabry-Perot formalism (in this case for a rectangular waveguide), i.e. the condition for a wave within the layer is that

$$2nd\cos b = p\lambda$$

where b is the angle between the beam and the normal of the surface. If we express this condition using the angle α between the beam and the layer surface, we get

$$2nd\sin\alpha = p\lambda$$

We see that p = 0 corresponds to light traveling straight ahead, while larger values of p correspond to larger angles.

The difference between the angles is given by

$$2nd\cos\alpha\,\Delta\alpha = \Delta p\lambda = \lambda \quad\Rightarrow\quad \Delta\alpha \approx \frac{\lambda}{2nd}$$

for small angles. The thinner the layer, the larger the spacing between useful angles. If p = 1 corresponds to a larger angle than the one given by the numerical aperture, only the mode with p = 0 can propagate. In that case, we say that the waveguide works in single-mode.

The limiting thickness for this to occur is obtained by putting p = 1 for nsinα = NA:

$$\lambda \quad = \quad 2nd \sin \alpha \quad = \quad 2d_{\text{limit}} NA \qquad \Rightarrow \qquad d_{\text{limit}} \quad = \quad \frac{\lambda}{2NA}$$

In reality, the geometry is of course cylindrical and the problem is solved through expressing the wave equation in cylindrical coordinates:

$$\frac{\partial^2 E}{\partial r^2} + \frac{1}{r}\frac{\partial E}{\partial r} + \frac{1}{r^2}\frac{\partial^2 E}{\partial \phi^2} + \frac{\partial^2 E}{\partial z^2} - \left(\frac{n}{c}\right)^2 \frac{\partial^2 E}{\partial t^2} \quad = \quad 0$$

In this formula, two problems immediately arise, of which the first one has to do with the polarization. It will play in, because it actually breaks the cylindrical symmetry, since the light reflected as TE and the light reflected as TM will have different phase shifts upon reflection, respectively. We choose to disregard from this problem in a first approximation, since it mostly concerns higher order modes.

Let the wave be expressed as

$$E(r,\phi,z,t) = R(r) e^{im\phi} \, e^{i(\omega t - \beta z)}$$

The angular part must look like this because it has to be periodic, with the circumference as the period. We assume here that m is an integer.

The z-dependence looks like it does because we assume that the wave consists of a propagating part in the z-direction with an effective k-value of β and a standing part in the radial direction. The assumption is based on experience from the plane waveguide. If this assumption gives solutions describing the reality, these obviously have a certain value.

OK, insertion gives now

$$e^{im\phi} \, e^{i(\omega t - \beta z)} \left(\frac{\partial^2 R}{\partial r^2} + \frac{1}{r}\frac{\partial R}{\partial r} \right) - \frac{\mathrm{Re}^{i(\omega t - \beta z)}}{r^2} m^2 e^{im\phi} - R\beta^2 e^{im\phi} \, e^{i(\omega t - \beta z)} + k^2 R \beta^2 e^{im\phi} \, e^{i(\omega t - \beta z)} = 0$$

It does not look that pleasant, but can be simplified dividing by the factor

$$e^{im\phi} \, e^{i(\omega t - \beta z)}$$

We now obtain an equation:

$$\frac{\partial^2 R}{\partial r^2} + \frac{1}{r}\frac{\partial R}{\partial r} - \left(k^2 - \beta^2 - \frac{m^2}{r^2} \right) R \quad = \quad 0$$

With joy one immediately realizes that this is the Bessel equation (true?).

The general solution consists now of Bessel functions of the first kind, with order m. Choosing specifically m = 0, i.e. the fundamental mode, we obtain that the solution is a zeroth order Bessel function with hr as argument, where

$$h = \sqrt{k^2 - \beta^2}$$

In the fiber core h is real, which leads to Bessel functions of the first kind. Physically this can be interpreted as β being the average component of k in the propagation direction. For the fundamental mode, h can be expected to have a value close to zero. The reason for it not being identically zero is due to the fact that diffraction generates all the time components perpendicular to the axis.

In the fiber cladding, h becomes imaginary for the values of β corresponding to propagating modes in the core. The reason for this is that we obviously have to use the cladding index to calculate k, leading to real values of h in the core and imaginary values in the cladding, for an interval of values of β.

If h is real in the cladding, too, this corresponds to a radiating mode, i.e. for which the condition of total reflection is not fulfilled.

We now know that m is an integer label for one of the two mode numbers. The other one is obtained upon (with the chosen value of m) determination of β from the condition that R must be continuous across the interface between core and cladding.

This must be solved graphically, the interested reader can find the details in Yariv: Optical Electronics.

An important conclusion is nevertheless that hr can be maximally 2.405 for the 1st mode, 5.52 for the 2nd etc.

This gives us a condition for only the zeroth mode being able to propagate, namely

$$\frac{2\pi}{\lambda}\sqrt{n_1^2 - n_2^2}\, r_{core} < 2.404 \quad \Rightarrow \quad r_{core} < \frac{0.383\lambda}{NA}$$

usually called the single-mode condition.