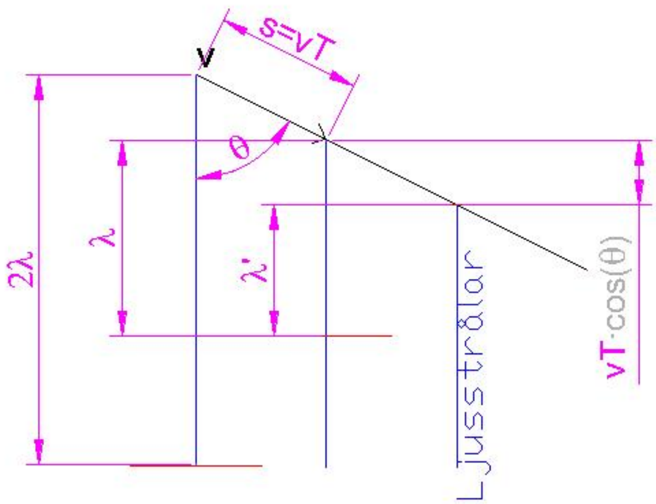
Optical flow- and velocity measurements

In optical measurements of flow, a laser-based Doppler method called LDV (Laser Doppler Velocimetry) is often used. The method is based on the Doppler effect, derived below for a source moving with velocity v, emitting light with a wavelength λ in the frame of reference of the source, equaling c/f.



The particle is viewed at three consecutive moments of light emission, separated in time by one period of the light. The distance traveled by the particle is consequently s = vT.

At each one of these moments a light front is emitted. The first one of them has thus traveled a distance of 2λ and the second one a distance of λ when the third front is being emitted. However, since the emitting particle has moved between these emissions the distance between the wavefronts will be

$$\lambda' = \lambda - vT\cos\theta \quad \Rightarrow \quad \Delta\lambda = -vT\cos\theta$$

Since we are going to measure a beat frequency between this light and light from particles in rest, it is advantageous to express this in terms of the frequency shift:

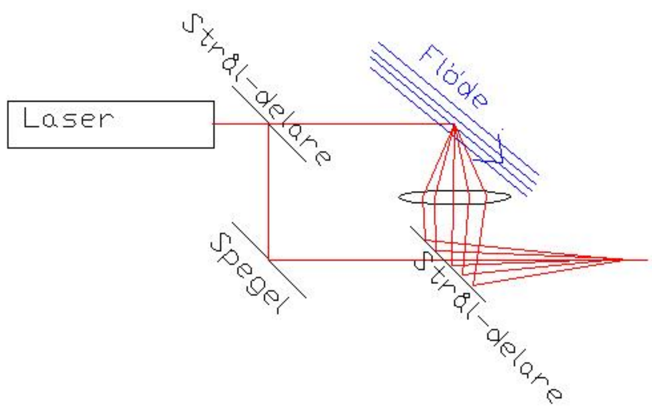
$$\Delta f = \Delta\left(\frac{c}{\lambda}\right) = -\frac{c}{\lambda^2}\Delta\lambda = \frac{c}{\lambda^2}v\frac{\lambda}{c}\cos\theta = \frac{v}{\lambda}\cos\theta$$

In the case of light not emitted by the particle, but rather light being reflected from it, this can be generalized to

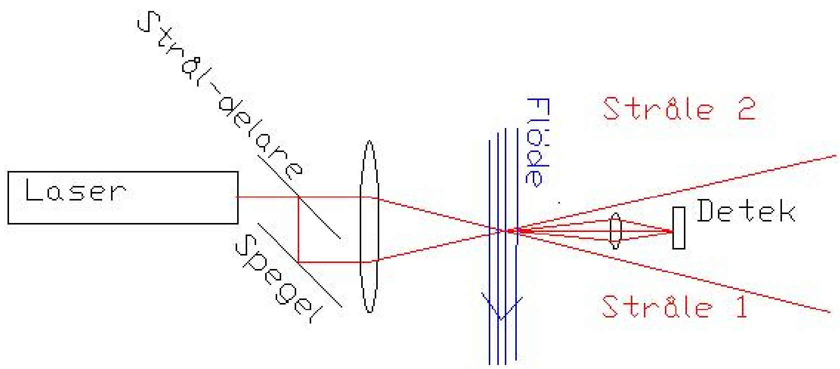
$$\Delta f = \frac{v}{\lambda}\left(\cos\theta_{in} - \cos\theta_{out}\right)$$

where the angles are those between the incoming light and the velocity vector and the scattered light (the direction of observation) and the velocity vector, respectively.

One could possibly think that it would be interesting to see that this expression can be derived from the relativistic expression for time dilation, which also gives that the relative change of the frequency is v/c. This sounds like a technically impossible measurement, but nevertheless it is exactly this factor that makes it possible to determine the sought velocity from the measurements.
The idea is to let the scattered light interfere with light not being scattered by the moving target and therefore conserving its original frequency/wavelength. One can expect that the intensity is varying as a function of the difference frequency through interference (beats).
There are two principally different experimental set-ups, of which the first one is illustrated in the figure below.



As one can see, this is a set-up resembling a Mach-Zender interferometer, with the "small" difference that one mirror is replaced by the streaming liquid. Scattering in a liquid is essentially isotropic, only a minor amount of the incoming light being scattered. The first beamsplitter must consequently transmit almost all the light and reflect only a very small portion.
The scattered light must then be collected with a lens onto a detector, which means that light scattered in different directions will be gathered. This leads to a corresponding uncertainty in velocity determination since the cosine factors will vary between different directions. One comes to a somewhat unpleasant trading-off situation between light economy and measurement precision. For this reason, method 2 was developed early, working according to the figure below:



The idea is not having an unaffected reference beam, but rather letting the scattered light from two laser beams interfere.
The scattered light from beam 1 gets the frequency

$$f_1' = f + \frac{v}{\lambda}\left(\cos\theta_{in,1} - \cos\theta_{out,1}\right)$$

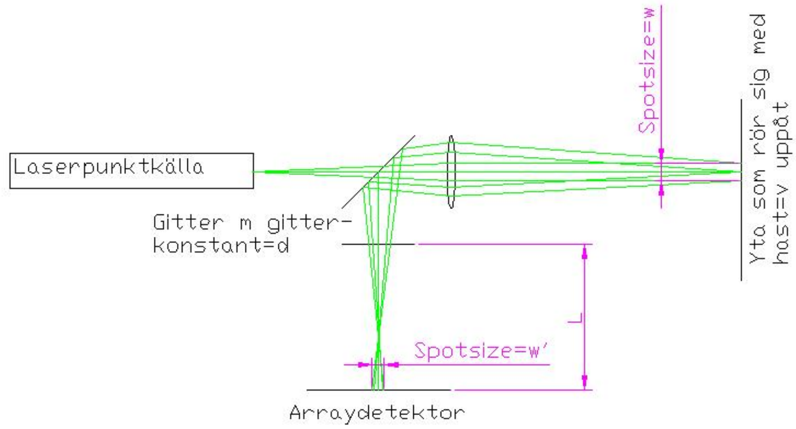
where $\theta_{in,1}$ is the angle between beam 1 and the flow, and $\theta_{out,1}$ is the angle between the flow and the scattered light. This is the same for the scattered light from beams 1 and 2, respectively, which is why we can omit the indices 1 and 2 in the scattered beam.
The beam 2 is treated analogously

The beat frequency at the detector is now

$$\Delta f = f_1' - f_2' = \frac{v}{\lambda}\left(\cos\theta_{in,1} - \cos\theta_{in,2}\right)$$

i.e. it does not depend on which one of the scattered beams is being recorded since all the pairs give the same difference frequency.

A numerical example: Let the beams 1 and 2 form an angle of 30^{o} with the symmetry axis of the lenses. This gives a trigonometric factor of 1. Choosing a 514 nm wavelength (Ar-ion laser) we get a difference frequency of about 2 MHz for a flow velocity of 1 m/s. Electric frequencies up to about 10 GHz are measurable, which will give us the measurement range.
The weakness of the method is that a very strong laser is required since the fraction of the light scattered in the small solid angle subtended by the detector lens is small. It is furthermore only possible to use the method to study movements in solid materials and laminary flow in fluids.
Turbulent flow gives results impossible to interpret.
An alternative to above in velocity measurement of passing surfaces is to focus light on the surface and then look at the surface reflex, imaged by a lens and a grating. The set-up will be like in the figure below:



The laser (usually of semiconductor type) illuminates the surface of a moving object through a lens with spot diameter w. This surface is imaged back through the same lens on a plane between a grating and an array detector. On the array detector a (defocused) spot with diameter w' is seen, tripled because of the grating (three grating orders).
This means that a detail passing through *w* during time *t* will pass through *w'* during time (*w'/w*)*t*, which means that that the detail (or a shadow or a diffraction pattern) of it is moving with the velocity

$$v' = v\frac{w'}{w}$$

The distance to the detector plane, *b*, of the three copies of the same detail is given by the grating formula

$$b = L\tan\theta \approx L\sin\theta = \frac{L\lambda}{d}$$

The time of passage of the same detail shadow on the detector element will be

$$t = \frac{b}{v'} = \frac{L\lambda w}{dvw'} \Rightarrow \quad frequency = v\frac{dw'}{\lambda Lw}$$

Of course, many different frequencies are being recorded by the detectors because the surface is more or less dull, generating a noise-like signal. However, the above frequency is present all the time, so if the signal is passed through a frequency analyzer, it is easily measurable. This principle is used e.g. in optical mice.