# Photometry 1

The measurement of light often takes place in a system of units connected to the SI-system, but which is not based on the ”usual” units (meter, kilogram, second and ampere). The starting point is actually the radiant power, i.e. something that can be measured in watts (energy/time). This quantity is called the radiant flux.
To obtain the corresponding photometric unit, one multiplies the measured power at each wavelength with a weighing factor proportional to the sensitivity of a normal eye at this wavelength. This means that IR- and UV-radiations will obtain zero weight, and the green light will get the highest weight.

## Flux

The quantity we start with is the one corresponding to the radiant flux, consequently called the luminous flux.

(1) $\Phi_{\nu}$

This describes how much light

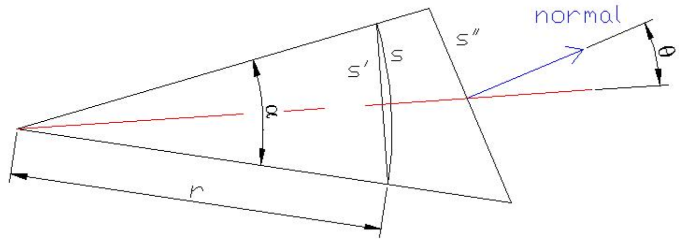
- Is in total emitted by a source
- Is in total received by a detector, a photographic film, the entrance pupil of a camera etc.
- Is transported through an optical system

However, it does not contain any information on the distribution of the flow on the receiving surface. Furthermore, it does neither inform on the size of the emitting source, nor on any angles obeyed by the flow.

Luminous flux from a lamp divided by the electrical power supplied to it is usually called the luminous efficacy and is measured in lumen/W.
For a conventional incandescent lamp this value is about 10 or slightly higher, for a low energy lamp at least 50, and for energy-efficient streetlights at least 100. The power not coming out as light is usually converted to heat or to heat radiation.
Furthermore, the luminous flux is what is called a conserved quantity, i.e. in the absence of losses the flux inserted into a system from one side comes out unaltered at the other side.
In order to describe the next quantity, the concept of solid angle is needed. If you are already familiar with this, you can jump to Luminous intensity.

# Solid angle

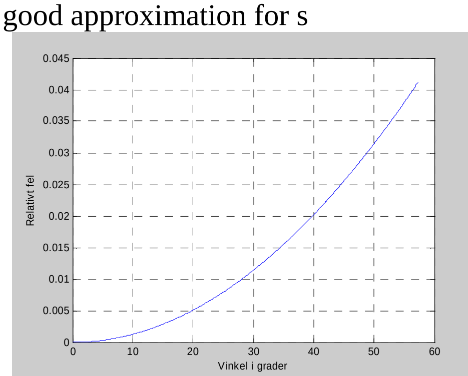
An ordinary (plane) angle is defined by connecting the rays with an arc, which has its center of curvature at the vertex.




The length of the arc is s and the plane angle α (measured in radians) is obtained as α = s/r.
This is the definition and not any approximation to small angles or similar.
For instance, for a right angle we obtain that the arc length is a fourth of the perimeter, i.e.

$$s = \frac{2\pi r}{4} \quad \Rightarrow \quad \alpha = \frac{s}{r} = \frac{\pi}{2}$$

which looks familiar.
If the angle is not large, the straight line s’ is a good approximation for s




However, if one wants to calculate the angle subtended by a tilted object with length s”, one has to multiply s” with the cosine of the tilt angle.
In a corresponding way we now want to define an angle extending in two directions. The meaning of this is not intuitively clear, but also in this case we define the angle as the quotient of a measure of the size of the object and a function of the distance.
For reasons, which will become obvious, one has chosen to divide the area of the part of a sphere limited by the rays (which will be many and are called generatrices) by the square of the radius of this sphere.
We thus define the solid angle measured in steradians as

$$\Omega = \frac{S}{r^2}$$

This can also be approximated for small angles (see above) by the corresponding plane surface S’.
The solid angle of a tilted surface is obtained by multiplying by the cosine of the tilt angle.
Some interesting special cases are a small circular surface and a hemisphere:

$$\Omega_{hemisphere} = \frac{2\pi r^2}{r^2} = 2\pi \; ster \qquad \Omega_{small\ circle} = \frac{\pi (r\alpha)^2}{r^2} = \pi\alpha^2$$

## Luminous intensity

Luminous intensity is a property of the source and describes how the flow is distributed in different directions.
The unit for luminous intensity is 1 cd (which is the same as 1 lumen/steradian) and is essentially the starting point for the photometric units (but it would be pedagogically foolish to start with it).

Definition:

$$I_v = \frac{\Phi_v}{\Omega} = \frac{d\Phi_v}{d\Omega}$$

i.e. the smaller the solid angle a given flux is directed into, the larger will the luminous intensity be in that direction. The unit 1 cd corresponded originally to an extremely well-defined candle light, and even if its definition has been changed to a more general one, 1 cd still gives a good description of a candle.
A 60 W light bulb with a luminous efficacy of 12 lumen/W thus has the luminous intensity of 57 cd.

If it is placed in a reflector, directing the light into a cone with a half top angle of $10^{\circ}$, one gets 7600 cd.
Think a moment whether you understood how I got the above results. If it was not clear, maybe the chapter on solid angles could be of some help.

## Illuminance

The quantity one most often comes in contact with is the illuminance, which is defined as the flow per area. We will in the following treatment designate receiving surfaces as S, S’, S” etc. and emitting surfaces as A, A’, A” etc.
The definition and a very useful corollary:

$$E_v = \frac{d\Phi_v}{dS} = \frac{d\Phi_v}{d\Omega}\frac{d\Omega}{dS} = I_v\frac{\cos\theta}{r^2}$$

The last one is most often much more useful than the definition itself.
From this follows for instance that the illuminance under a street light emitting 12 000 lm into a hemisphere, gives a luminous intensity of 2 000 cd below it, and consequently an illuminance given by

$$E_v = I_v\frac{\cos^3\theta}{h^2}$$

where h is the height of the post and θ is the angle out to the illuminated point. One can see for instance that the illuminance goes down to half the value when

$$\cos^3\theta \quad = \quad 1/2 \qquad \Rightarrow \qquad \theta \quad = \quad 37.5^o$$

## Luminous emittance

Is a quantity used less often, but needed in calculations. It gives the emitted flow of the light source per source area, but does not inform on the direction the light is emitted to.
Definition:

$$M_v \quad = \quad \frac{\Phi_v}{A} \quad = \quad \frac{d\Phi_v}{dA}$$

The second equality is used when the surface is radiating unevenly.
The quantity is being used when describing the brightness of a backlight, but above all when estimating how bright a surface will be for a given illumination.
If a surface scatters a fraction of $R_{diffus}$ (called diffuse reflectance), a surface illuminated by E will obtain a luminous emittance of

$$M_v \quad = \quad R_{diffuse} E_v \qquad R_{diffuse} \in [0,1]$$

## Luminance

The last one of our quantities is the most useful one, but also the most hard to explain in words: Luminance is the luminous intensity in a given direction divided by the projected area in that direction.
As we will see in the chapter on photometry in imaging systems, it is the luminance of the object that will determine how light it will appear.
Definition and a simple corollary:

$$L_v \quad = \quad \frac{\Phi_v}{\Omega A} \quad = \quad \frac{I_v}{A}$$

For a plane, ideally dull surface (= Lambert surface) the luminous intensity will decrease according to

$$I_v(\theta) \quad = \quad I(\theta = 0)\cos\theta$$

i.e. the luminous intensity of a plane surface decreases when looking at it in an angle. Furthermore, the projected area also decreases, which gives

$$L_v \quad = \quad \frac{I_0 \cos\theta}{A\cos\theta} \quad = \quad \frac{I_0}{A}$$

i.e. it is independent on the angle of observation.
“An illuminated dull wall looks equally illuminated, regardless whether one is looking at it obliquely or from straight ahead”

## Relation between luminance and illuminance

How large will $I_0$ be, expressed in terms of the incoming flow?

$$\Phi_v = \iint_{hemisphere} I_0 \cos\theta \, d\Omega = \iint_{hemisphere} I_0 \cos\theta \sin\theta \, d\theta \, d\varphi = I_0 \pi$$

Understanding this beautiful integral is not necessary for accepting the following important formula

Dividing this relation by π and the source area, one obtains

$$L_v = \frac{\Phi_v}{\pi A} = \frac{M_v}{\pi} = R_{diffuse} \frac{E_v}{\pi}$$

This is a very powerful relation with the help of which for instance the visual impression of a cloth illuminated by a projector can be calculated.